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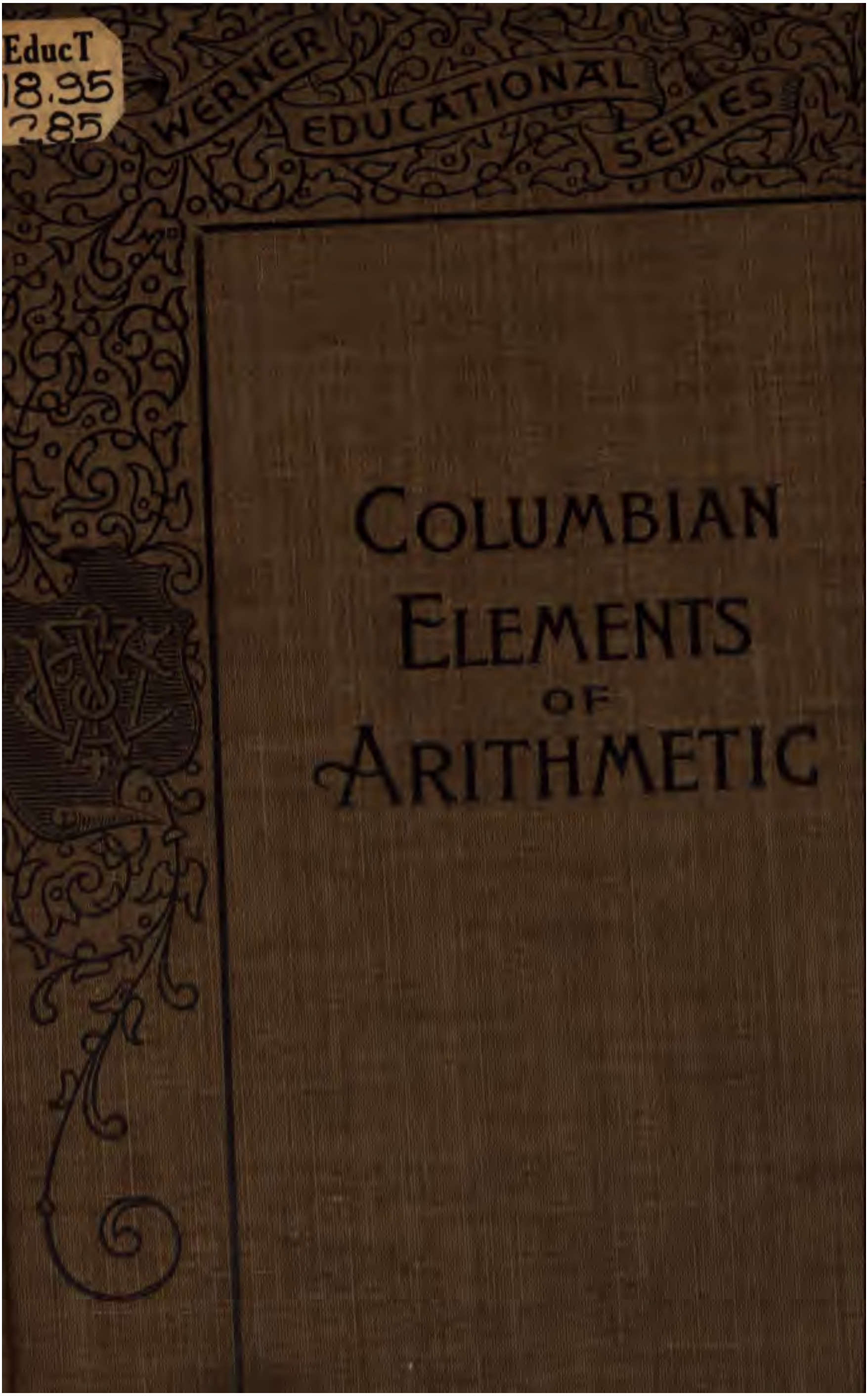

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Received May 15, 1903.

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Elements of Arithmetic

Oral and Written

FOR

Lower and Middle Grades

Revised and Enlarged

CHICAGO NEW YORK
THE WERNER COMPANY

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PREFACE.

A Text-Book in any branch of study should be an aid to teacher and pupil. Its matter should be graded, and its form should be simple, logical and systematic. It should unfold from some fundamental idea around which all its essential facts should be grouped.

Arithmetic is a means of training the *will*, since the material world is *controlled* by numbers. The intellect may think the world, but cannot change any of its forms. By his will man asserts his dominion over matter and arithmetic is the branch of study which gives him this mastery.

The material world is made to serve man's purposes by *counting*, *weighing*, *measuring* and *valuing* it. Man's wealth, whether in cattle, acres, wares or dollars, is determined by *counting*. The products of field, workshop and mine are weighed, measured and valued. Arithmetic is therefore a practical study which deals with material things. The land on which we dwell, the houses in which we live, the clothing which we wear and the food which we eat, is determined and brought within our control through numerical processes. Arithmetic is an active force in our civilization, and its study is of fundamental importance.

Number is a property of things, but numbers are expressions for things. Hence it is plain that if the child is to gain the mastery of the art of using numbers, he should derive his ideas from *counting*, *weighing*, *measuring* and *valuing* the things which come within his experience. The imagination rather than the memory should be exercised in learning the arithmetical art.

The elements of arithmetic, as detailed in this book, are unfolded from this point of view. The child is led forward by easy steps to grasp arithmetical facts and afterward to use numbers to express them. His work is first *oral through objects*, and after the chief ideas are fixed with small numbers, he is required to deepen his knowledge through written work.

The method presented here for the use of pupils of the first three years is synthetical as developed in the St. Louis Schools. Thought is awakened through material things, numbers are applied orally and by means of figures, and thus the child thinks and learns to express his thought in logical form.

The analyses, rules and explanations are merely directive to aid in obtaining good form, but the live teacher will find that he can use the good things he has acquired without being hampered by a parade of "words without knowledge," which so often prove to be serious hindrances of advancement.

NOTE TO ENLARGED EDITION.

The present work has been thoroughly revised and enlarged by the addition of much concrete work which the publishers feel will surely add to its value and usefulness as a text book. The subjects of Fractions and Decimals have been changed while that of Compound Numbers has been entirely re-written and greatly increased. Percentage, Interest, Bank Discount and kindred subjects, and Ratio and Proportion have been introduced and treated in a simple and objective manner. Few books of its character contain a better or more carefully graded class of examples, which fact admirably adapts it not only to graded but also to ungraded and district schools.

The publishers acknowledge their obligations to Mr. J. W. Hall of the St. Louis Schools, for editorial work in the revision of this edition.

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Course of Study.

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Elements of Arithmetic.

1. **Arithmetic** is the art of using numbers.

2. **Numbers** are expressions for things.

3. Numbers are **expressed** by means of ten characters, called **Figures.**

Name:	*One*	*Two*	*Three*	*Four*	*Five*	*Six*	*Seven*	*Eight*	*Nine*	*Cipher*
Print:	1	2	3	4	5	6	7	8	9	0
Script:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*

PRACTICAL DRILLS.

4. Teach children to count up to 10.

To the Teacher:—Teach numbers *first*, and use figures to express them after they are learned. In imparting the first ideas of number, a numeral frame should be used. *Distinguish counting from mere numbering.* One thing is one. One and one *put together*, is two. Three ones, or two and one *put together*, is three. By taking four, five, six, seven, eight or nine things, and *putting them together*, the idea of four, five, six, seven, eight or nine is imparted, or arises in the mind. Use various kinds of objects with which to make the first ideas of number clear. Remember that the mind imposes number upon things.

MAKING NUMBERS.

5. Teach children to make, realize and express all numbers up to 10.

Two consists of—

Two ones | | = || $1+1=2$.

The sign + (**plus**) is read **and.** The sign = (**equality**) is read **is** or **are.**

Three consists of—

Three ones, | | | = ||| $1+1+1=3$; or,
Two and one, || | = ||| $2+1=3$.

Four consists of—

Four ones, | | | | = |||| $1+1+1+1=4$,
Two twos, || || = |||| $2+2=4$, or,
Three and one, ||| | = |||| $3+1=4$.

Five consists of—

Five ones, | | | | | = ||||| $1+1+1+1+1=5$,
Two twos and one, || || | = ||||| $2+2+1=5$,
Three and two, ||| || = ||||| $3+2=5$; or,
Four and one, |||| | = ||||| $4+1=5$.

Six consists of—

Six ones, | | | | | | = ||||||
$1+1+1+1+1+1=6$,
Three twos, || || || = |||||| $2+2+2=6$,
Two threes, ||| ||| = |||||| $3+3=6$,
Four and two, |||| || = |||||| $4+2=6$; or,
Five and one, ||||| | = |||||| $5+1=6$.

Seven consists of —

Seven ones, | | | | | | | = |||||||
$1+1+1+1+1+1+1=7$,
Three twos and one, || || || | = ||||||| $2+2+2+1=7$,
Two threes and one, ||| ||| | = ||||||| $3+3+1=7$,
Four and three, |||| ||| = ||||||| $4+3=7$,
Five and two, ||||| || = |||||||. $5+2=7$; or,
Six and one, |||||| | = ||||||| $6+1=7$.

Eight consists of —

Eight ones, | | | | | | | | = ||||||||
$1+1+1+1+1+1+1+1=8$,
Four twos, || || || || = |||||||| $2+2+2+2=8$,
Two threes and two, ||| ||| || = |||||||| $3+3+2=8$,
Two fours, |||| |||| = |||||||| $4+4=8$,
Five and three, ||||| ||| = |||||||| $5+3=8$,
Six and two, |||||| || = |||||||| $6+2=8$; or,
Seven and one, ||||||| | = |||||||| $7+1=8$.

Nine consists of —

Nine ones, | | | | | | | | | = |||||||||
$1+1+1+1+1+1+1+1+1=9$,
Four twos and one, || || || || | = |||||||||
$2+2+2+2+1=9$,
Three threes, ||| ||| ||| = ||||||||| $3+3+3=9$,
Two fours and one, |||| |||| | = ||||||||| $4+4+1=9$,
Five and four, ||||| |||| = ||||||||| $5+4=9$,
Six and three, |||||| ||| = ||||||||| $6+3=9$,
Seven and two, ||||||| || = ||||||||| $7+2=9$; or,
Eight and one, |||||||| | = ||||||||| $8+1=9$.

PRIMARY COMBINATIONS.

6. Teach pupils to add any two small numbers.

2's

1	2
1	0
2	2

3's

2	3
1	0
3	3

4's

2	3	4
2	1	0
4	4	4

5's

1	2	5
4	3	0
5	5	5

6's

1	2	3	6
5	4	3	0
6	6	6	6

7's

1	2	3	7
6	5	4	0
7	7	7	7

8's

1	2	3	4	8
7	6	5	4	0
8	8	8	8	8

9's

1	2	3	4	9
8	7	6	5	0
9	9	9	9	9

10's

1	2	3	4	5	10
9	8	7	6	5	0
10	10	10	10	10	10

11's

1	2	3	4	5	11
10	9	8	7	6	0
11	11	11	11	11	11

12's

1	2	3	4	5	6	12
11	10	9	8	7	6	0
12	12	12	12	12	12	12

13's

1	2	3	4	5	6	13
12	11	10	9	8	7	0
13	13	13	13	13	13	13

14's

1	2	3	4	5	6	7	14
13	12	11	10	9	8	7	0
14	14	14	14	14	14	14	14

NOTATION TO THOUSANDS.

7. **Teach children that nine and one make ten, and that two figures are required to express ten (10).**

Figures are used to spell numbers, just as letters are used to spell words. Thus: *a*, *an*, *ant*, *Nat*, and *tan*, are spelled by combining the three letters, *a*, *n* and *t*, variously; so, by means of the figures, 1, 2 and 3, the numbers 1, 12, 123, 213 or 312 are expressed.

Nine and one is ten, which requires the two figures 1 (*one*) and 0 (*cipher*) to express. As *o* and *n* will spell either *on* or *no*, so 1 and 2 will express either 12 or 21. The relative position in which the ten figures are placed makes it possible to spell all numbers with ten characters.

☞ Fix this thought in the child's mind as early as possible — **Figures are the Alphabet of Arithmetic.**

8. **Numbers increase ten-fold from right to left. Thus:**

Units....................	1	2	3	4	5	6	7	8	9
Tens	10	20	30	40	50	60	70	80	90
Hundreds	100	200	300	400	500	600	700	800	900
Units, Tens and Hundreds:	111	222	333	444	555	666	777	888	999

9. Teach numbers from 10 up to 100, and that all numbers between 9 and 100 consists of UNITS and TENS.

A numeral frame with a hundred balls is needed for this purpose. The right-hand figure expresses **units**, and the next figure on the left, **tens**. Thus: three tens and seven units is expressed by writing three to the left of seven — 37, which is called thirty-seven. *Count off tens and units on the numeral frame, and then express the number in figures.* Require children to count by tens and units, until they realize in a general way the numbers from 9 to 99, and can represent them in figures.

10. Require children to read numbers from 1 to 99 written promiscuously and to realize them by means of the numeral frame.

Exercises.

11	21	31	41	51	61	71	81	91
24	34	44	53	64	74	84	94	14
37	47	56	67	77	87	97	27	17
42	52	62	72	82	92	12	22	32
55	65	75	85	95	15	25	35	45
68	78	88	98	18	28	38	48	58
73	83	93	13	23	33	43	53	63
86	96	16	26	36	46	56	66	76
99	19	29	39	49	59	69	79	89
30	40	50	60	70	80	90	10	20

11. Teach numbers from 99 to 1000, and that all numbers between 99 and 1000 consist of UNITS, TENS and HUNDREDS.

☞ Three figures constitute a **period.**

Exercises.

121	565	919	672	126	787	233
234	678	341	785	239	892	346
347	783	454	898	560	915	459
452	896	567	913	674	128	780

In reading numbers, do not use the word **and.** Thus: 896 is read: eight hundred ninety-six.

FUNDAMENTAL PROCESSES.

12. Numbers may be increased or diminished.

1. **Addition** is the process of finding a number by putting two or more given numbers together.

2. **Subtraction** is the process of finding a number equal to the difference between two given numbers.

3. **Multiplication** is the process of finding a number which is a given number of times another number.

4. **Division** is the process of finding how many times one number is contained in another of the same kind.

5. **Division** is also the process of separating a number into parts of a given size, or of finding the size of given parts. This is the case whenever a concrete number is divided by an abstract number.

Since the fundamental processes are intimately connected, one growing out of the others, arithmetical ideas are best developed by taking each digit in order and teaching all the processes in connection with it.

13. Teach all the processes with ONE.

1. Count from 1 to 20.
2. Count backwards from 20.
3. Give some number and let pupils tell what number is one more or one less than the given number. Thus: Six is one more than *five* and one less than *seven*.

Given numbers: 8, 3, 9, 2, 5, 7, 4, 11, 10, 12.

☞ Teach that a number multiplied or divided by **one**, is unchanged.

Problems.

1. A boy spent one cent for a top, one cent for a pencil, one cent for paper, and one cent for a sponge; how many cents did he spend?

2. John has three cents, which is one cent more than George has; how many cents has George?

3. If apples are worth a cent a piece, how much will eight apples cost?

4. At a cent a piece, how many marbles can be bought for nine cents?

5. A man divided seven peaches equally among seven boys, how many peaches did each boy receive?

☞ Make similar problems and drill upon them until each process is fixed and pupils can give a *fluent* analysis. Let pupils make problems. *See pp.* 20–21.

14. Teach all the processes with TWO.

1. Count by twos from 1 to 29.
2. Count by twos from 0 to 30.
3. Count by twos backward from 20.

☞ Take small objects such as peas, grains of corn, and separate into twos, and count backwards by taking two away repeatedly.

4. Count backwards by twos from 11.

5. Give some number and let pupils give the numbers which are two more or two less than the given number. Thus: 7 is 2 more than 5 and 2 less than 9.

Given numbers: 9, 13, 5, 16, 8, 4, 11, 17, 6, 15.

6. Add 2 to the digits, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

$2+2=$ $1+2=$ $3+2=$
$4+2=$ $5+2=$ $6+2=$
$8+2=$ $7+2=$ $9+2=$

☞ Drill upon these combinations until each pupil can give results accurately and promptly.

7. Add 2 to all numbers from 9 to 20.

☞ Pay special attention to 11+2; 12+2; and 13+2.

A good **device** to help the child realize numbers at this stage, is the use of **plinths** an inch square with the figure 1 pasted upon them, and two inches long and an inch wide with the figure 2 pasted upon them. With these he may **build** up his column and realize how numbers grow out of **repetition.**

DRILL CARDS.

Take narrow strips of **pasteboard** and write twenty **numbers** promiscuously arranged upon them. The arrangement of the numbers on each card should differ as much as possible. **Distribute** the cards to the class and let each pupil, as rapidly as possible, **add 2** to the **numbers** on his card.

8. Subtract 2 from all numbers between 1 and 12.

For the Blackboard.

2	5	8	11	4	7	3	9	6	10
2	2	2	2	2	2	2	2	2	2
—	—	—	—	—	—	—	—	—	—

Subtraction grows out of addition and consists of finding complementary numbers. Thus: 12—2 involves the thought of 2 and *what* = 12. Hence—

2 and *what* = 5	2 and *what* = 8
2 " " = 9	2 " " = 7
2 " " = 6	2 " " = 3
2 " " = 10	2 " " = 4
2 " " = 12	2 " " = 9

The process of subtraction is an act of comparison in which the mind sees that the difference between two numbers is a complementary number, which, when added to the less number, makes the greater number,

9. Multiply 2 by each digit.

Multiplication is a short process of adding the same number to itself any number of times. The child should derive his table from actual additions. Thus:

Table.	2	2	2	2	2	2	2	2	2
2× 1= 2	—	2	2	2	2	2	2	2	2
2× 2= 4	2	—	2	2	2	2	2	2	2
2× 3= 6		4	—	2	2	2	2	2	2
2× 4= 8			6	—	2	2	2	2	2
2× 5=10				8	—	2	2	2	2
2× 6=12					10	—	2	2	2
2× 7=14						12	—	2	2
2× 8=16							14	—	2
2× 9=18								16	—
2×10=20									18

10. Division is the process of measuring one number by means of another. Thus: take eight blocks and separate these into groups of two each, viz.: II II II II. We find that there are four groups of 2 each. This may be expressed in two ways: $8 \div 2 = 4$, or $\begin{array}{l}2\overline{)8}\\ \ \ 4\end{array}$

Now separate the eight blocks into two equal groups: IIII IIII. Each group will consist of 4. Separating a number into two equal parts is finding one-half of it. $\frac{1}{2}$ of $8 = 4$.

Exercises.

1. Seperate the following numbers into groups of 2 each: 10, 14, 18, 12, 20, 16, 4, 6.

2. Find one-half of each of the following numbers: 10, 6, 14, 16, 12, 18, 8, 20, 4.

Problems.

1. John spent 2 cents for a pencil, 2 cents for paper, 1 cent for a rubber and 2 cents for a stamp; how many cents did he spend?

Analysis.—Since John spent 2 cents for a pencil, 2 cents for paper, 1 cent for a rubber and 2 cents for a stamp, he spent, in all, the **sum** of 2 cents + 2 cents + 1 cent + 2 cents, or 7 cents.

2. Mary has 7 cents which amount is 2 cents more than Jane has; how many cents has Jane?

Analysis.—Since the 7 cents which Mary has are 2 cents more than Jane has, Jane has as many cents as the **difference** between 7 cents and 2 cents, or 5 cents.

3. How much will 8 lemons cost at 2 cents each?

Analysis.—Since 1 lemon costs 2 cents, 8 lemons will cost 8 times 2 cents, or 16 cents.

4. If 1 orange costs 2 cents, how many oranges can be bought for 18 cents?

Analysis.—If 1 orange can be bought for 2 cents, as many oranges can be bought for 18 cents, as 2 cents are contained times in 18 cents, which is 9 times. Therefore, at 2 cents each, 18 cents will purchase 9 oranges.

5. If 14 apples are divided equally between 2 boys, how many apples does each boy receive?

Analysis.—Since 14 apples are divided equally between 2 boys, each boy receives ½ of 14 apples, or 7 apples.

SUGGESTIONS.

Addition.—Place pencils in the right or left hand of a boy and let him count them and tell how many pencils he has in his hand. (*I have 5 pencils in my right hand.*) Let other children count and tell how many pencils he has in his right hand. (*Harry has 5 pencils in his right hand.*) Place 2 other pencils in his left hand and let him tell the number of pencils he has in his left hand. (*Harry has two pencils in his left hand.*) Let Harry now tell the number of pencils he has in each hand. (*I have 5 pencils in my right hand and 2 pencils in my left hand.*) Let the children tell the number of pencils Harry has in each hand.

By skillful questioning get some member of the class to ask a question in regard to the number of pencils Harry has in both hands. (*How many pencils has Harry in both hands?*)

The next step is to combine these elements so as to make a problem. (*Harry has 5 pencils in his right hand and 2 pencils in his left hand; how many pencils has he in both hands?*)

☞ Take other objects and drill until each member of the class can make a problem with facility, after which teach the proper form of analysis. *Be careful that each pupil uses* **correct language.** DEVELOP THE PARTS OF A PROBLEM AND THEN COMBINE THE PARTS. *Let them write a problem after it is made.* Ask, in what is this problen? (*This is a problem in* **addition of 2's.**)

Subtraction.

Put 7 pencils in one hand and take 2 pencils away, and ask, *What did I do?* (*You put 7 pencils in one hand and then took 2 pencils away.*) Elicit the proper question: (*How many pencils are left in your hand?*) Have a child put several pencils in his hand, and then take 2 away and ask how many are left in his hand. (*I had 7 pencils in one hand and took 2 away; how many pencils remain in my hand?*)

☞ Make other problems in **subtraction of 2's.**

Multiplication.

Call out 9 girls and give 2 pencils to each of them. Ask, *What did I do?* (*You gave 2 pencils to each one of 9 girls; how many pencils are given to all of them?*)

Division.

1. Take 12 pencils in your hand and call out a number of children. Give 2 pencils to each until all are given away, and send the other children to their seats. Ask, *What did I do?* (*You gave 2 pencils to each pupil?*) Elicit the proper question: (*To how many children did you give 2 pencils each?*) Combine into a problem: (*I had 12 pencils and gave 2 of them to each of several children; to how many children did I give pencils?*)

2. Take 14 pencils and call 2 girls. Divide the pencils equally between them. Ask, *What did I do?* (*You divided 14 pencils between 2 girls.*) Elicit the proper question. (*How many pencils did I give to each girl?*) Combine into a problem: (*I had 12 pencils and divided them equally between 2 girls; how many pencils did I give to each girl?*)

Exercises.

1. Mary has 2 pencils in each hand, how many pencils has she?

2. How many are 5 pencils and 2 pencils?

3. How many are 9 pencils less 2 pencils?

4. If George has 16 pencils, how many can he give to each of his two brothers?

5. How many 2's are there in 16? in 18?

6. Mary has 13 apples and gives 2 to her sister, how many has she left?

7. 13 less 2 are how many? 17 less 2?

8. If a man buys 12 bushels of coal, how much will he have after he burns 2 bushels?

9. If 1 yard of cloth is sold for 2 dollars, how many yards of the same kind could be bought for 18 dollars?

15. Slate and Blackboard Work.

1. Write upon the blackboard columns of 1's and 2's each, also of 1's, 2's and 0's mixed, and secure rapid addition.

2. Let the pupils add the same columns beginning with other numbers; as 3, 7, 11.

3. Dictate numbers for pupils to write as high as hundreds, using only 0's, 1's and 2's.

Note.—In writing numbers from dictation, require pupils, when called upon in turn, to first say the number and then spell it figure by figure. Thus: Two hundred twelve, two–one–two; two hundred one, two–cipher–one. This fixes the number in his mind and cultivates a careful habit that will be beneficial later on.

4. Carefully inspect all work done by pupils and see that the numbers are written in vertical columns. Let all the improper work be rewritten.

Note.—Teachers should carefully explain the value of units, tens and hundreds and the method of carrying in addition.

Exercises.

Dictate the following numbers for addition.

1. 212; 112; 200; 100; 222; 101; 212; 112; 101; 221; 222;
2. 112; 102; 210; 100; 221; 111; 101; 212; 220; 122; 121;
3. 220; 212; 115; 202; 121; 122; 111; 221; 102; 220; 115.
4. 122; 221; 122; 211; 120; 202; 112; 111; 212; 222; 221.

☞ Make similar exercises.

5. Teach the process of subtraction.

The number from which another number is to be taken, is called the **Minuend.** The number which is to be taken from the **minuend,** is called the **Subtrahend.** The complement of the subtrahend is called the **Difference.**

Minuend: 71632170271
Subtrahend: 2221211212
Difference: 69410959059

Explanation.—Whenever an order of the subtrahend is greater than the corresponding order of the minuend, add the value (10) of a higher unit to the order of the minuend, and 1 to the next order of the subtrahend. Thus: 2 is greater than 1. Add ten units to the one of the minuend, making it 11 units, subtract 2 units from 11 units which leaves 9 units and add one ten to the one ten of the subtrahend. So in every similar case.

☞ The addition of ten to any order of the minuend and one to the next order of the subtrahend is a *practical device* by which the contradiction of taking a greater quantity from a less quantity is overcome. It is simply the adding of *equals* to unequals, which does not change the difference.

Examples for Dictation.

(1)	(3)	(5)
92015341721	7101572141	71029161421
22212212	2221212	121112212

(2)	(4)	(6)
801271251271	50127518211	6012845131
2212212212	222122112	22221212

Do not **read** the numbers but **spell** them by dictating the figures in **line** from left to right. Tell pupils under what figure to begin to write the subtrahend. Begin to subtract on the right.

6. Teach the process of multiplication, using all the digits.

The number to be multiplied is called the **Multiplicand.** The number by which the multiplicand is taken, is called the **Multiplier.** The result of the multiplication is called the **Product.**

Example.

Multiplicand: 2120221211212
Multiplier: 8

Product: 16961769689696

Explanation.—8 times 2 units is 16 units, one ten and six units. Write the six units and add the ten to the result when one ten is taken 8 times. Proceed in the same manner with each order of units.

7. Write a long number on the blackboard and let pupils multiply it by each of the digits.

Example for Practice.

2121122021212122
(1,2,3,4,5,6,7,8,9)

8. Teach the process of division, using 2 as a divisor.

The number to be divided is called the **Dividend.** The number by which the division is made, is called the **Divisor.** The number showing how many times the divisor is contained in the dividend, is called the **Quotient.** The part of the dividend which remains after the division is completed, is called the **Remainder.** The remainder is usually expressed in a fractional form.

Example.

Divisor 2) 71925813726391 **Dividend**

Quotient 35962906863195$\frac{1}{2}$. **Ans.**

Explanation.—Begin on the left. 2 is contained in 7 three times and 1 over which makes ten units of the next lower order. Ten added to 1 makes 11. 2 is contained in 11 five times and 1 over. Proceed in this manner until the units are reached and then write the 1 which remains over the divisor 2.

Exercises

(1)

2) 917283746591

(2)

2) 8317938265471

(3)

2) 82917325863143

(4)

2) 641275693184273

When numbers are dictated for subtraction, multiplication, or division, spell them by giving a figure at a time to be written in line.

9. Teach the half of multiples of two.

Any number of times a number is called its **Multiple.** Thus: the multiples of 2 are 4, 6, 8, 10, 12, and so on.

Exercises.

1. How much is two and the half of two?

2. How much is eight and the half of eight?

3. How much is twelve and the half of twelve?

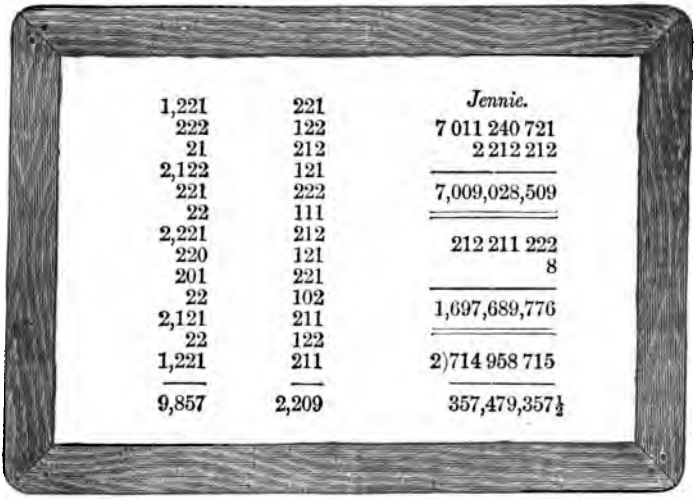
4. How much is six and the half of six?

5. If a dot increases the value of a *musical* note one half, what is the value of a dotted half-note? whole-note? quarter-note?

16. Teach the form of a written examination.

Take the slate with its longer side toward you. Do not write too near the edges nor crowd your work on the slate. Give five examples and mark any one of them 0, if there is even one figure wrong. Secure neat work.

Model Slate.



1,221
222
21
2,122
221
22
2,221
220
201
22
2,121
22
1,221
9,857

221
122
212
121
222
111
212
121
221
102
211
122
211
2,209

Jennie.
7 011 240 721
2 212 212
7,009,028,509

212 211 222
8
1,697,689,776

2)714 958 715
357,479,357½

☞ **Make similar examinations and drill on this work until at least 90% of accuracy is obtained.**

Problems.

1. A farmer pastured thirteen droves of sheep during the season, containing respectively 212, 121, 202, 111, 221, 200, 211, 222, 122, 101, 220, 110, 221 sheep; how many sheep did he pasture in all?

2. Mr. A owns 70,221 acres of land, which is 2,112 acres more than Mr. B owns; how many acres does Mr. B own?

3. How much would 212,212 barrels of flour be worth at $7 a barrel?

4. A grocer received $7,194 for apples at $2 a barrel; how many barrels of apples did he sell?

5. A man bequeathed to his two sons $9,462; how much was each to receive?

6. There are 7 birds in one cage and 2 in another; how many birds in both cages?

7. John had 12 marbles and lost 2; how many had he left?

8. What will 9 turkeys cost at 2 dollars a piece?

9. If a man spends 20 dollars for books, at 2 dollars a piece; how many does he get?

10. If 2 trees yield 18 bushels of apples, how many bushels does one tree yield?

11. From a piece of cloth containing 13 yards, 2 yards were cut off; how many yards remained?

12. If a boy earns 2 dollars in one week, in how many weeks can he earn 16 dollars?

13. John has 6 apples and his brother gave him 2 more; how many had he then?

14. There are 2 pints in a quart. How many pints are there in 8 quarts?

15. If 2 sacks of flour are worth 8 dollars, what is one sack worth?

16. How many pieces each, when 2 children get 20 pieces of candy?

17. There are 2 boys in a seat and 6 seats in a division; how many boys in the division?

18. If a man can travel 7 miles in one day and 2 miles the next day, how far does he travel in both days?

19. A grocer bought 9 bushels of apples and sold 2 bushels; how many bushels had he left?

20. If one barrel of apples is worth 2 dollars, how many barrels can be bought for 24 dollars?

21. Willie has 6 rabbits and Henry has 2; how many have both?

22. If there are 2 peaches on a dish, how many would there be on 7 dishes?

23. If 2 sheep were sold for 20 dollars, what was the price of each?

24. John has 8 marbles and his brother has 2 more, how many has his brother?

25. William has 2 marbles and his brother has 6 times as many, how many has his brother?

26. A man earns $2 per day, how much will he earn in a week?

27. James gives each of his six brothers 2 apples, how many does he give them all?

28. In a class there are 9 boys and two more girls than boys; how many girls in the class?

29. If Mary has 2 rings on each finger of her right hand, how many rings has she?

30. If I divide 18 apples among 2 girls, how many apples will each girl receive?

31. If a man divided $16 between his 2 children, how many dollars will he give to each?

32. If 2 children sit at each desk, how many children sit at 8 desks.

33. If 1 yard of ribbon cost $2, how much will 9 yards cost?

34. How many balls at 2 cents each can be bought for 10 cents?

17. Teach all the processes with THREE.

1. Count by threes from 0 to 30.
2. Count by threes from 1 to 28.
3. Count by threes from 2 to 29.
4. Count by threes backward from 28, from 29, and from 30.

By using small objects or the numeral frame and separating the THREES, these processes can be sensibly illustrated. Since number is a property of things, conscious ideas of number are primarily obtained from the objects themselves.

5. Give some number and let pupils tell what other number is *three* less or *three* more than the given number. Thus: Nine is *three* less than 12 and three more than 6.

Given numbers: 4, 21, 8, 13, 6, 15, 7, 25, 27, or 19.

6. Add 3 to the digits.

$1+3=$ $2+3=$ $3+3=$

$4+3=$ $5+3=$ $6+3=$

$7+3=$ $8+3=$ $9+3=$

7. Add 3 to all numbers from 3 to 30.

1, 11, and 21; 2, 12, and 22; 3, 13, and 23 should be associated.

☞ **Use the drill cards.**

8. Subtract 3 from all numbers from 3 to 12.

For the Blackboard.

10	7	6	4	9	5	7	11	8	12
3	3	3	3	3	3	3	3	3	3
—	—	—	—	—	—	—	—	—	—

As the process of the addition of two numbers is a process by which the mind puts two numbers together and gets a new number, so subtraction is a process in which the mind sees complementary numbers. To develop this idea, let such questions as these be asked:

3 and what = 7
3 " " = 11
3 " " = 5
3 " " = 9
3 " " = 12

3 and what = 8
3 " " = 4
3 " " = 10
3 " " = 6
3 " " = 3

9. Multiply 3 by each digit.

The child should derive his table from actual additions, since multiplication is simply a short process of adding equal numbers.

	3	3	3	3	3	3	3	3	3
Table.	—	3	3	3	3	3	3	3	3
3×1= 3	3	—	3	3	3	3	3	3	3
3×2= 6		6	—	3	3	3	3	3	3
3×3= 9			9	—	3	3	3	3	3
3×4=12				12	—	3	3	3	3
3×5=15					15	—	3	3	3
3×6=18						18	—	3	3
3×7=21							21	—	3
3×8=24								24	—
3×9=27									27

☞ **Figures should not be used until the numerical ideas which they express are clearly apprehended.**

10. Find how many times 3 is contained in all numbers up to 30.

Give children a number of objects more than three and less than thirty. Let them count the objects and then separate them into groups of three. Then let them count the threes and tell how many threes and how many remain over.

Separate 18 into groups of 3 each, viz.: ||| ||| ||| ||| ||| |||. We find that there are 6 groups of 3 each. This may also be expressed in two ways:

$$18 \div 3 = 6; \text{ or } \begin{array}{r} 3\overline{)\,18} \\ 6 \end{array}$$

Now separate the 18 into three equal groups: |||||| |||||| ||||||. Each group will consist of 6.

$\frac{1}{3}$ of 18 = 6, and $\frac{2}{3}$ of 18 = 12.

Exercises.

1. Separate the following numbers into groups of 3 each: 9, 15, 6, 18, 21, 30, 27, 24, 12.
2. Find one-third of each of the following groups: 12, 9, 27, 6, 15, 30, 18, 24, 21, 27.
3. Find two-thirds of 12, of 18, of 9, of 15, of 21, of 6.

Problems.

1. A boy spent 3 cents for an orange, 2 cents for taffy, 1 cent for an apple, 3 cents for nuts, 1 cent for candy and 3 cents for peaches; how much did he spend in all?

2. John has 9 marbles which number is 3 more than George has; how many marbles has George?

3. How much will nine tops cost at 3 cents each? at 2 cents each? at 1 cent each?

4. How many tops will 24 cents buy at 3 cents each? at 2 cents each?

5. If 27 apples are divided equally among three boys, how many apples will each boy receive?

Analysis.—Since there are three boys, each will receive $\frac{1}{3}$ of 27 apples, or 9 apples.

6. If 18 oranges are divided among three girls, how many oranges will each receive?

7. Divide 21 dolls among three girls.

18. Slate and blackboard work.

Write a column of 3's, also of 0's, 1's, 2's and 3's mixed, upon the blackboard and drill upon rapid addition. Also reverse the process by subtracting the numbers of the column successively. Also multiply the numbers of the column by the digits 1 to 9, reading as rapidly as possible. Also divide numbers below 30 by the numbers of the column, giving remainders. Find halves and thirds of all multiples of 2 and 3 below 30.

Note.—The pupil ought by this time to be moderately accurate in writing units, tens and hundreds. Teach them that these three places constitute the units' period, and that the units' period is separated from the thousands' period by a comma.

At first only dictate thousands and require the pupil to say the number indicating the thousands followed by a comma, and then wait for the dictation of the units' period. Thus:

dictating 3,205. Teacher says ”three thousand “, Pupil „3 - comma “ Teacher, “two hundred five”. Pupil “2–0–5.”

In like manner dictate the following:

1. Find the sum of 3,012; 313; 31; 2,123; 330; 32; 3,232; 301; 32; 3,203; 32.

2. From 71,025 subtract 21,031.

3. Multiply 31,323 by 1, 2, 3, 4, 5, 6, 7, 8, 9.

4. Divide 71,927 by 3, by 2, by 1.

5, Find the sum of 1,321; 232; 3,213; 2,331; 223; 1,303; 1,202; 2,013; 1,230; 323; 231; 323; and 232.

6. From 91,027 take 33,222.

7. Multiply 21,032 by 2, 3, 4, 5, 6, 7, 8, 9.

8. Divide 62,893 by 2, and by 3.

9. Find the sum of 2,013; 1,302; 2,131; 3,012; 2,013; 3,120; 231; 3,012; 2,012.

10. From 91,862 take 22,012.

19. Teach ⅓ and ⅔ of multiples of 3.

11. How much is ⅓ of 18? ⅔ of 18?

12. How much is ⅓ of 24? ⅔ of 24?

13. How much is ⅓ of 21? ⅔ of 21?

14. How much is ⅓ of 27? ⅔ of 27?

15. How much is ⅓ of 12? ⅔ of 12?

Example.—What is ⅔ of 18.

Analysis.—Since ⅓ of 18 is 6, ⅔ of 18 is two times 6, which is 12.

Problems.

1. William bought a cow for $ 30.21, a sheep for $ 2.33, a plow for $ 22.30, harness for $ 33.21, a cart for $ 23.10, a harrow for $ 13.32, and five hogs for $13.30 ; how much did he pay for what he bought ?

There is no good reason why children at this stage should not be taught how to **write dollars and cents.** They can thus deal with values within the limits indicated.

2. If a farmer still owes on his farm $ 2313.22 and has paid $926.49 ; how much did his farm cost him ?

3. A's farm is worth $ 9173.61, which is $ 2023.23 more than B's farm is worth ; what is the value of B's farm ?

4. What is the value of 3230 acres of land at $ 8 an acre.?

5. How many barrels of apples at $ 3.00 a barrel can be bought for $ 6187.00?

6. Three partners gained $ 986. 37; what was each man's share of the gain ?

7. How many barrels of flour can you buy for 12 dollars at 3 dollars a barrel ?

8. A lady bought a hat for 3 dollars, a shawl for 2 dollars, and a pair of gloves for 2 dollars. How much did she pay for all ?

9. A man earns 18 dollars in 3 weeks ; how much is that a week ?

10. How many trees in my orchard, if there are 7 rows and 3 trees in each row ?

11. There are 3 feet in one yard. How many feet are there in 6 yards ?

12. A boy caught 11 bass and sold 3 ; how many does he keep?

13. A grocer sold 7 pounds of sugar to one man, 2 pounds to another and 3 pounds to another; how much did he sell to the three men?

14. A lady bought a dress for 3 dollars; how much change should she receive from a ten-dollar bill?

15. At 3 dollars a pair, how many pairs of shoes can be bought for 18 dollars?

16. If three sheep cost 15 dollars, how much does one cost?

17. How long will it take a boy to earn 21 dollars, if he earns 3 dollars a week?

18. A grocer has three soap boxes. In one there are 14 bars, in another 3 bars and in another 2 bars. How many bars are there in these boxes?

19. There are 21 days in 3 weeks; how many days in one week?

20. Five lead pencils will cost how much at 3 cents each?

21. On a bill of 7 dollars I have paid 3 dollars; how much do I yet owe?

22. A man bought a sheep for 3 dollars and sold him for 7 dollars; how much did he gain?

23. A butcher bought 8 sheep at one time, 3 at another time, and 2 at another time; how many did he buy in all?

24. At 3 cents a yard, what will 9 yards of calico cost?

25. There are 9 bushels of apples in 3 barrels; how many in each barrel?

26. How long will it take to go 12 miles, if I go 3 miles an hour?

27. A boy bought a book for 10 cents, a slate for 3 cents, and a lot of pencils for 2 cents. How much did he pay for all?

28. How many apples at 2 cents each can be bought for 14 cents?

29. I sold a quantitv of wool for 9 dollars and received in part payment a barrel of flour at 3 dollars. How many dollars remained due?

30. If three books cost 21 dollars, how much does one book cost?

31. When oranges are 3 cents a piece, how much must you pay for six?

32. Emma had 15 cents and her mother gave her 3 cents more, how much had she then?

33. There are 27 pencils in boxes. If there are 3 pencils in each box, how many boxes are there?

34. Two men floated 12 miles down the river in a skiff, and then rowed 3 miles up again. How far were they from the starting place?

35. Mr. Brown pays 3 dollars a week for board. How much does it cost him at the end of 8 weeks?

36. There are 24 persons living in 3 houses; how many persons in one house?

38. A farm contains 43 acres of woodland, 20 acres of meadow, 32 acres of pasture and 33 acres of plowed land; how many acres in the farm?

39. If a man earns $32 a month, how much will he earn in 7 months?

40. Take 223 oxen from 2006 oxen.

41. A man's property worth $41,184, was divided equally among 3 persons; what was each one's share?

42. At $3 a ton, how many tons of ice can be bought for $1,569?

20. Teach all the processes with FOUR.

1. Add 4 to all numbers from 1 to 20.

2. Subtract 4 from all numbers from 4 to 20.

3. Count 40 or more by fours beginning with 0, 1, 2. 3.

4. Reverse the operation and beginning with 40, 41, 43, take away successively, 1, 2, 3, 4.

5. Multiply 4 by each digit and develope the table, as in 2's and 3's.

6. Find how many 4's in all numbers up to 40.

7. Find $\frac{1}{4}$ of all multiples of 4 up to 40.

8. Write a column of 4's, also of 0's, 1's, 2's, 3's and 4's mixed on the blackboard and drill on addition, subtraction, multiplication and division. Encourage accuracy and rapidity and prevent repetition.

9. Dictate numbers containing ten-thousands, using no figure greater than 4 and require pupils to spell the numbers, as in (pp. 22-23) and lay special stress on the fact that the units' period must contain 3 places. Require pupils to rewrite improperly arranged work.

Exercises.

1. Add 21,042; 43,424; 41; 4,214; 41,243; 4,042; 22,402; 4,043; 404; 42,311; 4,002; 420; 24,044.

2. From 82,732 take 34,424. From 71,431 take 24,304.

3. Multiply 41,231 by 2, 3, 4, 5, 6.

4. Multiply 24,134 by 6, 7, 8, 9, 4, 3, 5, 2.

5. Divide 62,435 by 2, 3, 4. 6. Divide 72,341 by 2, 3, 4.

7. Add 41,004; 4,243; 4; 23,041; 42,334; 4,243; 41,124; 4,241; 43,424; 44,241; 2,414; 40,040; 404.

8. From 98,102 take 43,314. 9From 81,605 take 34,142.

Problems.

1. A lad spent 9 cents for nuts, 3 cents for paper, 4 cents for pencils, 2 cents for pens, 4 cents for candy, 3 cents for apples, 2 cents for a sponge, and 4 cents for marbles; how many cents did he spend in all?

2. Henry has eight cents which sum is four cents more than Julia has; how much money has Julia?

3. How much will seven oranges cost at 4 cents each? at 3 cents? at 2 cents? at 1 cent?

4. How many lemons will 32 cents buy at 4 cents each?

5. If a man distribute 28 peaches equally among 4 boys; how many peaches does each boy receive?

6. How many pounds of rice can I buy for 24 cents at 4 cents a pound?

7. A boy has 12 marbles in one pocket, 3 in another and 4 in his hand; how many has he with him?

8. A girl had 12 pins and lost 4 of them; how many had she left?

9. At 4 cents each what will 7 oranges cost?

10. If 4 books cost 24 dollars, what will one book cost?

11. If 3 barrels of flour cost 21 dollars, what will one barrel cost?

12. A farmer sold 6 dozen eggs to one man, 4 dozen to another, 3 dozen to another; how many dozen did he sell?

13. If one pencil costs 2 cents, what do 8 pencils cost?

14. At 4 cents each, how many papers of pins can be bought for 32 cents?

15. A farmer having 2,700 bushels of corn, fed 1,342 bushels to his cattle; how many bushels had he left?

16. Henry has 4 chinas, Charley has 2, August has 3; how many have all?

17. John owes his brother 6 dollars, and has paid him 4 dollars; how much does he still owe him?

18. If 3 hats cost 12 dollars, what does one hat cost?

19. A lady paid 32 dollars for board at the rate of 4 dollars a day. How many days did she board?

20. If one yard of ribbon costs 4 cents, what do 9 yards cost?

21. If four bushels of wheat make 1 barrel of flour, how many barrels will 9,652 bushels make?

22. A man filled 20 sacks of flour putting 9 pounds into each sack; how many pounds of flour did he use?

23. A man having 451 acres of land, gave 322 acres to his son; how much land remained?

24. A farmer sold a span of horses for $314, four oxen for $420, and some cows for $234; how much did he receive?

25. A teamster hauled 4,701 bricks in three loads; how many bricks in each load?

26. Mr. Smith borrowed $9,180 and has since paid $2,413; how much remains due?

27. There are six books on a table. The first contains 311 pages, the second 434 pages, the third 243 pages, the fourth 144 pages, the fifth 430 pages, and the sixth, 224 pages. How many pages in the six books?

28. An officer divided 8,880 pounds of food among some soldiers, giving 4 pounds to each man; how many men were there?

29. What is the cost of 434 barrels of flour at $9 a barrel?

30. A carpenter is paid $344 for 4 months' work; how much does he get per month?

31. I paid $2,400 for my farm, $134 for my horse and cart, $24 for my garden utensils, $54 for a mowing machine, $40 for a horse-rake, and $104 for a yoke of oxen. How much did I pay for the farm, stock and implements?

32. In a corn field containing 4 acres, there are 19,360 hills of corn; how many hills in 1 acre?

33. If a man can walk 4 miles in an hour, how long will it take him to walk 124 miles?

34. There are 8 quarts in a peck; how many quarts in 244 pecks?

35. Mr. A has 343 sheep in 2 lots. In 1 lot there are 134; how many are there in the other?

36. A lady bought 7 yards of ribbon at 4 cents a yard; how much did it cost?

37. There are 12 bushels of apples in 4 barrels; how many bushels in each barrel?

38. An agent sold 13 houses for the following prices: $4,231.42; $34,243.21; $3,241.23; $4,420.40; $31,244.14; $404.24; $2,401.42; $42,412.23; $24,240.22; $33,424.14; $4,232.24; $44,230.40; $21,242.44; how much did he receive for all of them?

39. A farmer bought a farm for $94,231.22, and paid on it $43,323.43; how much does he still owe?

40. A coal company sold 424,234 tons of coal at $7 a ton, how much did they receive for the coal?

41. At an average of $4 a head, how many hogs can be bought for $9,724?

42. A man bequeathed to his four children $76,428.32; how much did each of them receive?

5. If the products of a manufactory for five years are worth $9,262,461, what is the average value per annum?

6. $7,524 are divided among 5 men; what is each man's share?

7. Three boats are used to carry soldiers. 1,245 men are on the first boat, 2,200 on the second, and the third carries 1,554 men. How many men on the three boats?

8. Two men raise 5,194 bushels of wheat. One of them raises 2,325 bushels of it; how much does the other raise?

9. Three horses cost $720; what was the price of each?

10. If it costs $4,235 to grade one mile of railroad, what did 6 miles cost?

11. A farmer gave $13 dollars for a cart, $5 for a plow, and $4 for a harrow; how much did they cost him?

12. What is the cost of 12 vests at $5 each?

13. Three men earned $27; how much did each man earn?

14. Of a regiment of soldiers containing 1,230 men, 544 were killed in battle; how many men were left?

15. Mr. Jones paid $450 for a lot, built a house on it for $5,340, a stable for $345, some fences for $135, and paid $43 for planting some trees. How much did the whole place cost him?

16. With 2 horses at each car, how many street cars can be hauled by 372 horses?

17. Mr. Thompson's farm contains 143 acres. How much land would there be in a farm six times as large?

18. If five horses cost $1,500, what is the cost of one of them?

19. How many oranges at 5 cents each can be bought for 20 cents?

20. How many dollars in eight purses containing $5 each?

21. A boy bought a sled for 20 cents and paid 5 cents for repairing it. How much had it cost him then?

22. I bought 524 cords of wood from Mr. A, 353 cords from Mr. B, 245 cords from Mr. C, and 154 cords from Mr. D; how many cords did I buy?

23. A farmer having 923 acres, willed 435 acres to his son, Charles, and the remainder to William. How much did William receive?

24. How many teams at $5 a day can be hired for $145?

25. In one piece of cloth there are 534 yards; how many yards in six such pieces?

26. If 1,104 acres be divided among four sons, how many acres does each receive?

27. If three baskets cost 30 cents, what is the price of each?

28. A lady went shopping with $24 and spent $5; how much had she left?

29. There are some wagons containing 45 persons. If there are 5 persons in each wagon, how many wagons are there?

30. Mr. A ownes 352 acres of land, Mr. B owns 425 acres, Mr. C 342 acres, and Mr. D 235 acres; how many acres are owned by these four farmers?

31. Supposing a city block is 542 feet long, how long are 8 blocks?

32. A cattle dealer having bought 3,240 sheep, shipped 1,250 on the cars, and sold the rest to farmers; how many did the farmers get?

33. A fruit dealer has 1,204 lemons in 4 boxes; how many in one box?

34. 2,415 presents are distributed at a school picnic, giving each child three articles; how many children are there?

35. A man bought some hay for \$10, some wood for \$4; how much more did the hay cost than the wood?

36. Three pairs of shoes cost \$15; how much is that a pair?

37. Charles is 13 years old, and is 5 years older than his sister Mary; how old is his sister?

38. A man sold 23 sheep from a flock consisting of 93; how many sheep remained?

39. A man paid \$545 for a span of horses, \$250 for a carriage, and \$54 for harness; how much did the whole cost him?

40. At \$5 per yard, how many yards of cloth can be bought for \$1,425?

23. Teach all the processess with SIX.

1. Add 6 to all numbers from 1 to 20.

2. Subtract 6 from all numbers from 6 to 20.

3. Count 60 or more by sixes, beginning with 0, 1, 2, 3.

4. Beginning with 60, 61, 62, 63, take away 6 successively.

5. Multiply 6 by each digit and develope the table of multiplication.

6. Find how many sixes in all numbers to 60, giving the remainders.

7. Find $\frac{1}{6}$ of all multiples of 6 to 60, also $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$ of all multiples.

8. Write a column of 6's, also a column of 0's, 1's, 2's, 3's. 4's, 5's and 6's mixed on the blackboard and practice addition, subtraction, multiplication and division.

9. Dictate numbers containing millions, reading only one period at a time and waiting until it is written.

Exercises.

1. Add 654,261; 5,345; 60; 30,402; 561; 6; 634,254; 65,602; 6,246; 663; 45; 63,400; 456,064.

2. From 732,594 take 632,656.

3. Multiply 562,616 by 6, 7, 8, 9, 4, 5.

4. Multiply 465,662 by 7, 4, 6, 9, 5, 8.

5. Divide 718,291 by 4, 5, 6.

6. Add 664; 546; 461; 653; 364; 256; 624; 563; 456; 521; 466; 654; 565.

7. From 536,415 take 265,456.

8. Divide 839,719 by 3, 4, 5, 6.

9. Multiply 632,636 by 8, 9, 7, 5, 6, 4.

10. From 935,674 take 546,655.

11. Add 56,402; 644,354; 6,346; 563,054; 6,326; 650; 62; 634; 502; 63,045; 544,463; 63,046; 44; 623,463.

12. Divide $16,344 equally among six men; how much does each man receive?

13. What is the cost of 4,335 tons of coal at $6 a ton?

14. A watch and chain cost $16. The chain cost $6; how much did the watch cost?

15. In an orchard are 36 trees in nine rows; how many in each row?

16. In a certain recitation 47 answers were given. If 6 of them were failures, how many were correct?

17. A man can carry six baskets at a time. How many times must he go to carry 672 baskets?

Problems.

1. Bought a slate for 6 cents, paper for 4 cents, pencils for 6 cents, a ruler for 5 cents, a copy book for 6 cents, and envelopes for 6 cents; how much was the cost of all?

2. One room contains 53 pupils which number is six more than there are in the next room; how many pupils are there in the next room?

3. How much will 12 copy books cost at 6 cents each?

4. If flour is worth $6 a barrel, how many barrels can be bought for $54?

5. If $48 will buy six hogs, what is the price of a hog?

6. There are 3 feet in a yard; how many yards in 9,747 feet?

7. At the rate of $6,556 each, what will it cost to build six houses?

8. A man's income is $59,159, and his expenses are $56,346; how much can he save?

9. In a certain library there are three book-cases. In the first are 356 books; in the second 626, and 654 in the third; how many books in the library?

10. What will 12 sheep cost at $6 a head?

11, At $6 each, how many hats can be bought for $96?

12. Mary had 69 cents and lost 6 cents; how much had she left?

13. Find the cost of 7 acres of land at $656 an acre.

14. At $4 a yard how many yards of broadcloth can be bought for $6,372?

15. At the beginning of a battle a General has 6,284 men. After the battle he had 4,425 men. How many did he lose?

16. A man divided his estate equally among six persons, giving all $11,688; how much did each one receive?

17. A man gave his property to his wife, two sons and three daughters. To his daughters he gave $1,554, to his wife $4,126, and to his sons, $6,456. How much did he give them all?.

18. A man bought some coal for $27 and some wood for $6; how much did it all cost?

19. How many yards of ribbon at 6 cents a yard can be bought for 84 cents?

20. A man having $41 gave $6 to his wife; how much did he have then?

21. What will be the yield of 8 acres of corn at the rate of 65 bushels an acre?

22. A carpenter built a house for $56,454, a stable for $45,643, and sheds for $45,645; how much did he receive for all the work?

23. Reading six pages a day, how many days will be required to read a book containing 372 pages?

24. Six men do a piece of work and receive $692,628 for it; how much is each man's share?

25. A dealer bought 562,494 sheep and sold 560,664; how many had he then?

26. If a man builds 48 rods of fence in 6 days, how much does he build in one day?

27. A merchant sold 6 cans of peaches from a box containing 91 cans; how many cans were left in the box?

28. What is the cost of 9 sheep at $6 each?

29. A miller wishes to put 923,764 bushels into 6 bins; how much shall he put into each bin?

30. Six men are worth $653,544 each; what are they all worth?

31. A man traveled 654 miles by railroad, 625 miles by stage, and 625 miles by steamboat; how far did he travel in all?

32. If 6 bushels of wheat make one barrel of flour, how many barrels of flour could be made from 787,866 bushels of wheat?

33. A man bought a piece of property for $976,348, and sold it for $65,636; how much did he lose?

34. Seven books at $6 each would cost how much?

35. Three cars have in them 36 persons; how many persons in each car?

36. How many bushels in a load containing 8 boxes, with 5 bushels in each box?

Abbreviations.

$.........dollars.
gal.......gallon.
bbl.......barrel.
lb...........pound.
rd............rod.
doz.........dozen.
ft............feet.
bu.........bushel.
ct. or c.......cents.

37. A dealer received an order for 2,763 bbl. of flour, but he had only 1,645 bbl. on hand; how many more bbl. did he require to fill the order?

38. Two boys started from the school-house. One went 53 rd. north, and the other went 45 rd. south; how far were they then apart?

39. At $6 a set how many toilet sets can be bought for $606?

40. A firm's expenses are $512, and the income is 7 times as much; how much is the income?

41. If a planter wishes to pack 1,324 lb. of cotton in three bales, how many lb. shall he put in each bale?

42. If a slate costs 7 ct. and a copy-book 5 ct., how much do both cost?

43. A steamer sailed 564 miles in 3 days; how far did she sail each day?

44. A man who had 1,204 yards of cloth, sold 321 yards; how many yards had he left?

45. In 6 bins there are 6,756 bushels of wheat; how many bushels in 1 bin?

24. Teach all the processes with SEVEN.

1. Add 7 to all numbers from 1 to 20.

2. Subtract 7 from all numbers from 7 to 20.

3. Count 70 or more by sevens, beginning with 0, 1, 2, 3, 4, 5, 6.

4. Beginning with 70, 71, 72, 73, take away 7 successively.

5. Multiply 7 by each digit and develope the table of multiplication.

6. Find how many sevens in all numbers to 70, giving the remainders.

7. Find $\frac{1}{7}$ of all multiples of 7to 70, also $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, of all multiples.

8. Write a column of 7's, also a column of 0's, 1's, 2's, 3's, 4's, 5's, 6's, and 7's mixed on blackboard and practice addition, subtraction, multiplication and division.

9. Dictate numbers containing millions, reading only one period at a time and waiting until it is written.

Exercises.

1. Find the sum: 21,017,746; 605,734,046; 61,074,306; 702,413; 60,126; 173,026; 67,677; 65,457; 63,407.

2. From 7,280,912 subtract 361,750.

3. Multiply 601,723 by 3, 4, 5, 6, 8, 7, 9, 2.

4. Add 2,463; 3,645; 5,746; 7,734; 5674; 3,746; 7,465; 5,675; 6,734; 7,365; 4,376.

5. Subtract 437,065 from 923,854.

6. Divide 725,836 by 6, 5, 7, 4, 3.

7. Multiply 647,352 by 5, 7, 8, 9, 6, 3, 4.

8. Take 506,743 from 935,827.

9. Divide 783,046,523 by 4, 5, 6, 7.

Problems.

1. What will be the cost of six oranges if each orange costs 7 ct.?

2. If you have 9 apples and buy 7 more, how many will you have then?

3· If a carriage wheel turns once in 7 seconds, how many times will it turn in 7,516,887 seconds?

4. A railroad's expenses for new cars were $151,567; for wages, $71,476; for clerk hire, $726,076; what was the amount of the expenses for that time?

5. A rope 89,565 ft. long is cut into 7 pieces; how long is each piece?

6. A grain elevator contains 7,107,567 bu. of wheat. How many more bu. must be put in to make it contain 9,131,894 bu?

7. If it takes 752,547 shingles for the roof of a house, how many shingles will be required for 9 such houses?

8. A farmer having 96 sheep sold 7 of them; how many were left?

9. At 7 ct. a yd. what is the cost of 9 yd. of ribbon?

10. At 7 ct. a piece how many copy-books can be bought for 91 ct.?

11. In one cornfield there are 751,207 hills, in the next there are 733,147 hills, and in the third there are 741,256 hills. How many hills in all these fields?

12. A tract of land containing 737,177 acres was divided into seven equal farms; how many acres in each farm?

13. How much money will a man have left from $1000 if he pays $747 for a horse?

14. If seven spoons are put into one box, how many boxes will it take to hold 21,742 spoons?

15. If the furniture in each of seven rooms in a residence is worth $7,137, what is it all worth?

16. Eight dollars and seven dollars are how many dollars?

17. At 7 ct. each how many apples can I buy for 84 ct.?

18. At 75 ct. each what will 7 song-books cost?

19. A and B buy a steamboat for $351,776, if A pays $72,175 how much does B pay?

20. On a train there are 721 passengers. In an accident one seventh of them were injured; how many were injured?

21. At 7 ct. a pound, how many pounds of honey can be bought for 73248 cents.

22. If a farmer puts his oats into 7 bins, each containing 257 bu., how many bu. has he?

23. At an election there were four candidates. The first received 7,324 votes, the second, 5,722, the third 21,007, and the fourth 30,727. How many votes were cast?

24. How much will 12 bbl. of pears cost at $7 per bbl.?

25. A girl gave 10 ct. for thread, 7 ct. for pins, 6 ct. for needles, and 7 ct. for some tape; how much did she spend?

26. A man expended $28 for sheep at $7 a head; how many sheep did he buy?

27. I have 72,737 sheets of paper out of which I wish to make 7 books. How many sheets must be used in each book?

28. At $7 each how many sheep can be bought for $198,604?

29. There are 735,726 bricks in a wall. How many bricks will be needed to build a wall 7 times as long?

30. A clerk whose income is $2,245 yearly, saves $1,707; how much does he spend?

31. If a train goes 75 mi. the first hour, 35 mi. the second hour, and 37 mi. the third hour; how far has it gone in the three hours?

32. A boy paid 45 cents for a reader, 47 cents for an arithmetic, 15 ct. for a slate, 7 ct. for a pencil, and 6 ct. for ink; how much did he pay for all?

33. Bought a house for $657, a lot for $276, parlor furniture for $267, a bed-room set for $265, kitchen utensils for $76, and dishes for $53; what was the cost of all?

34. A farmer sold some butter for 67 ct., eggs for 174 ct., lard for 77 ct.; how much did he receive for all?

35. John had 66 marbles, James 715, William 173, and Harry 65; how many had they in all?

36. In an orchard containing 77 trees there are 7 rows. How many trees in each row?

37. A man has a piece of work to do which 7 men do in 9 days; how long will it take him to do it?

38. Henry has 75 ct. given him by William and Thomas. William gave him 47 ct. How much did Thomas give him?

Abbreviations.

yd..........yard. mi..........mile. pk..........peck.
qt..........quart. T..........ton.

39. It takes 3 yd. of cloth to make a coat. How many coats can be made from 15,369 yd.?

40. If a man walks 7 mi. an hour, how many miles will he walk in 425 hours?

41. A man asked his neighbor where he was going with his 127 turkeys. He replied that he did not have 127, but if some one would give him 15, he would then have that number. How many did he have?

42. A lumber merchant sold his stock of lumber for $215,378, which was twice as much as it cost; how much did it cost?

25. Teach all the processes with EIGHT.

1. Add 8 to all numbers from 1 to 20.

2. Subtract 8 from all numbers from 8 to 20.

3. Count 80 or more by eights, beginning with 0, 1, 2, 3, 4, 5, 6, 7.

4. Beginning with 80, 81, 82, 83, take away 8 successively.

5. Multiply 8 by each digit and develope the table of multiplication.

6. Find how many eights in all numbers to 80, giving the remainders.

7. Find $\frac{1}{8}$ of all multiples of 8, also $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$ of all multiples.

8. Write a column of 8's, also a mixed column of the digits to 8 on the blackboard and practice addition, subtraction, multiplication and division.

9. Dictate numbers containing millions, reading only one period at a time and waiting until it is written.

Exercises.

1. Add 63,408,745; 7,380,694; 5,607,653; 404,075,806; 4,706,808; 32,806,756; 46,786; 46,780; 378; 78,675,486; 585,767,806.

2. From 856,980,794 take 372,685,437.

3. Multiply 75,834,267 by 3, 4, 5, 6, 7, 8, 9.

4. Add 4,786; 7,687; 5.678; 3,867; 5,608; 7,863; 7,865; 4,826; 7,683; 2,867; 6,758; 1,867.

5. Divide 786,756,978 by 3, 4, 5, 6, 7, 8.

6. Take 58,374,068 from 934,274,891.

7. Subtract 438,678,254 from 619,587,697.

8. Add 6,783; 4,786; 6,837; 5,867; 6,758; 4,873; 7,654; 8,774; 3,867; 4,567; 8,186; 4,764.

9. From 908,176,549 subtract 2,537,487.

Problems.

1. At $8 each how many sheep can be bought for $96?

2. A lady bought 8 qt. of milk costing 48 cents; what was the price per quart?

3. Mary had some peaches. She gave 8 to her mother, 7 to her brother, and had 8 left. How many had she at first?

4. If a train runs 38 miles in one hour, how far could it run in 8 hours at that rate?

5. If a gold ring costs $8, how many can be bought for $4,160?

6. A library contains 431,228 books. 82,185 volumes were destroyed by fire; how many remained?

7. A coal dealer sells 843,328 bu. of coal in July, 854,208 bu. in August, 845,358 bu. in September, and 830,418 in October. What did his sales amount to in the four months?

8. There are 8,568 sheep in a flock; if one-eighth of them should be sold, what number would remain?

9. A boy had eight cents, earned ten cents, and his mother gave him eight cents; how much had he then?

10. A newsboy selling papers at 5 ct. each, received 55 ct. How many did he sell?

11. What will be the expense of 8 weeks board at $9 a week?

12. There are 1,218 children in a school. Supposing each child uses eight pencils in a year, how many pencils are used by the school annually?

13. A letter-carrier has 878 letters to deliver in one day, and the next day he has but half of this number. How many does he carry the second day?

14. I have a string 1,516 ft. long. How much will remain if I cut off 828 feet?

15. At 8 ct. a pound, how many pounds of grapes can be bought for 816 cents?

16. One gross of Raub's Arithmetic cost 1,518 ct., one gross of Short's cost 2,408 cents, and a gross of Long's cost 2,008 cts. How much do the three gross cost?

17. If 4 pk. make a bu., how many bu. will 48 pk. make?

18. A commission merchant shipped 91 bbl. of fruit. Eight bbl. became spoiled. How many bbl. were good?

19. At 5 ct. a yd., what will be the cost of 9 yd. of ribbon?

20. An army of 318 men having plundered a city, took so much money that each man received $8. How much was taken in all?

21. There are 8,048 seats in a public hall, 3,508 of which are occupied; how many are vacant?

22. At $8 each how many pictures can be bought for $82,688?

23. In a certain army there are 43,458 cavalry, 8,548 artilerymen, and 824 engineers. How many men in the army?

24. On 8 acres of land 12,032 ears of corn were raised; how many ears were raised on one acre?

25. A boy at school made 64 failures in eight weeks; how many failures was that each week?

26. A farmer bought a wagon for $47, and after using it a month sold it at a loss of $8. How much did he get for it?

27. Mr. Williams paid $6 for a bbl. of flour, $8 for some sugar, $5 for potatoes, and $8 for some canned goods. What was the amount of the bill?

28. At a school picnic 3,108 children were at play. A shower came up and only 2,453 of them could get under shelter. How many got wet?

26. Teach all the processes with NINE.

1. Add 9 to all numbers from 1 to 20.

2. Subtract 9 from all numbers from 9 to 20.

3. Count 90 or more by nines, beginning with 0, 1, 2, 3, 4, 5, 6, 7, 8.

4. Beginning with 90, 91, 92, 93, take away 9 successively.

5. Multiply 9 by each digit and develope the table of multiplication.

6. Find how many nines in all numbers to 90, giving the remainders.

7. Find $\frac{1}{9}$ of all multiples of 90, also $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$, $\frac{8}{9}$ of all multiples.

8. Write a column of 9's, also a mixed column of the digits to 9 on the blackboard and practice addition, subtraction, multiplication and division.

9· Dictate numbers containing millions, reading only one period at a time and waiting until it is written.

Exercises.

1. Add 69,408,795; 7,897,694; 5,607,659; 493,085,896; 4,706,908; 32,896,750; 85,786; 96,789,379; 78,675,486; 65,879,786.

2. From 986,980,794 take 372,695,439.

3. Multiply 79,834,297 by 3, 4, 5, 6, 7, 8, 9.

4. Add 4,786; 7,689; 9,678; 3,867; 5,618; 7,893; 2,865; 9,426; 7,698; 3,867; 6,985; 1,969.

5. Divide 986,756,978 by 3, 4, 5, 6, 7, 8, 9.

6. Take 58,379,078 from 934,276,891.

7. Subtract 429,679,245 from 819,597,697.

8. Add 6,789; 9,786; 6,837; 5,897; 6,758; 4,873; 2,956; 8,754; 9,869; 8,186; 4,764.

9. From 908,179,549 subtract 84,537,997.

Problems.

1. There were 895,698 lb. of cotton on board a ship. After 45,248 lb. were unloaded, how many lb. remained on board?

2. A man has three houses. The first is worth $9,549, the second $5,919, and the third $3,598. What are they all worth together?

3. If one horse costs $289, what do nine horses cost?

4. $81 will buy how many pairs of boots at $9 a pair?

5. A farmer paid $15 for a cart, $9 for a plow, and $8 for a harrow; how much did he pay for these implements?

6, A grocer bought some potatoes for $90. Some of them having spoiled, he was compelled to sell them at a loss of $9. How much did he receive for them?

7. Nine men agree to build a house, each paying $4,218. What is the value of the house they intend to build?

8. There were 939 bbl. of apples at the depot. After 742 bbl. were shipped away, how many remained?

9. At $9 a cord how many cords of wood can be bought for $23,058?

10. In a factory there were employed 929 men, 98 women and 94 boys. How many persons are employed there?

11. Nine persons use 11,754 bbl. of flour in a year; how many bbl. must each one buy?

12. How many tops can be bought for 99 cents at 9 ct. each?

13. Henry has worked 12 examples, William 9, and John 8; how many examples were worked by these boys?

14. Find the cost of 91 T. of coal at $9 a ton.

15. How many lb. of flour at 4 ct. a lb. can be bought for 3,428 ct.?

16. In nine equal regiments there are 8,154 men. How many men in each regiment?

17. In a school where there are 791 children, 415 are girls. How many are boys?

18. If a train runs at the rate of 29 mi. an hour, how far will it run in nine hours?

19. How many pages in five books of which the first has 539 pages, the second 1,494 pages, the third 459, the fourth 1,439 and the fifth 1,913?

20. A little girl received 72 ct. from her mother, and her brother gave her one-ninth as much. How much did she receive from her brother?

21. In a class there are 17 girls, and 9 more girls than boys; how many pupils in the class?

22. A cattle dealer paid $47 for a cow, and sold her so as to gain $9; how much did he receive for the cow?

23. If yeast powder is put up in 8-lb. cans, how many cans will be needed to put up 2,568 lb.?

24. Nine lots of the same size were sold for $14,508. What was the price of one lot?

25. There is a farm consisting of 1,342 acres. 159 acres are sown in wheat. The remainder was in pasture land. How much is in the pasture?

26. There are 144 spools in 1 case, how many spools are there in 9 cases of that size?

27. In one week Henry won 48 chinas, John 59, and Tommy 95. How many did they all win that week?

28. A boy found a purse containing $63, and received one-ninth of the money for returning it to the owner. How much did he receive?

29. Bought 7 bbl. of apples at $9 a bbl.; what was the cost?

30. How much is lost if a carriage that cost $500 is sold for $375 ?

31. What will it cost to clear 789 acres of land at $9 an acre ?

32. The products of a farm during 9 years were valued at $19,467.89 ; what was their annual value?

33. At $8 a barrel, how many barrels of flour can be bought for $13,448.00?

34. The profits of a certain business were $1,847.63 last year, which is $5,324.74 less than it is this year ; what is the amount of this year's profits ?

35. The clearings of the New York banks were $47,374,869.72 during a certain period last year, which is $9,482,673.47 more than for the same period this year; how much are the clearings for the same period this year?

36. If a train of cars move 27 miles an hour, how far distant is a place which it requires 9 hours to reach ?

37. In how many hours can a steamboat go 1,764 miles, at the rate of 9 miles an hour?

38. How fast must a horse trot to go 40 miles in 8 hours?

39. If I invest $9,464 in flour at $8 a barrel and sell the flour at $9 a barrel ; how much would I gain?

40. A man bought flour at $8 a barrel and sold it at $9 a barrel and thereby gained $768 ; how many barrels of flour did he buy?

41. A thief is 78 miles ahead of his pursuers ; if they go 8 miles, while he goes 5 miles, how long will it take them to overtake him ?

27. Condensed Multiplication Table arranged in order of the Products.

4	2×2
6	2×3 3×2
8	2×4 4×2
9	3×3
10	2×5 5×2
12	2×6 6×2 3×4 4×3
14	2×7 7×2
15	3×5 5×3
16	2×8 8×2 4×4
18	2×9 9×2 3×6 6×3
20	2×10 10×2 4×5 5×4
21	3×7 7×3
22	2×11 11×2
24	2×12 12×2 3×8 8×3 4×6 6×4
25	5×5
27	3×9 9×3
28	4×7 7×4
30	3×10 10×3 5×6 6×5
32	4×8 8×4
33	3×11 11×3
35	5×7 7×5
36	3×12 12×3 4×9 9×4 6×6
40	4×10 10×4 5×8 8×5
42	6×7 7×6
44	4×11 11×4
45	5×9 9×5
48	4×12 12×4 6×8 8×6
49	7×7
50	5×10 10×5
54	6×9 9×6
55	5×11 11×5
56	7×8 8×7
60	5×12 12×5 6×10 10×6
63	7×9 9×7
64	8×8
66	6×11 11×6
70	7×10 10×7
72	6×12 12×6 8×9 9×8
77	7×11 11×7
80	8×10 10×8
81	9×9
84	7×12 12×7
88	8×11 11×8
90	9×10 10×9
96	8×12 12×8
99	9×11 11×9
100	10×10
108	9×12 12×9
110	10×11 11×10
120	10×12 12×10
121	11×11
132	11×12 12×11
144	12×12

Note.—The above table should be taught as early as when the pupils are on the 4's and carried on with the other digits, in which case such facts as that four 6's is the same as six 4's must be developed. Each step should be carefully developed objectively.

Thus: when developing $12\left\{\begin{array}{l}2\times6\\6\times2\\3\times4\\4\times3\end{array}\right.$ the teacher should place upon the blackboard 12 stars or other convenient marks, and show that they may be separated into six 2's or two 6's, also into four 3's or three 4's.

The recitation should be as follows:

Twelve equals six times two, or two times six, and four times three or three times four.

Uniformity in using the small factors first and observing that when the sign × is read times, the factor following should be read first, will add much to the exactness of the work.

Require the pupils to develope the table of multiplication of 10, 11 and 12, as in the digits.

28. Teach the use of 10 as a multiplier or divisor.

10 is a return to the **decimal scale.** By annexing a 0, a number is increased tenfold. Thus: $789\times10=7{,}890$. Dividing by 10 diminishes a number tenfold. Thus: $690\div10=69$.

29. Teach the use of 11 as a multiplier or divisor, (short division).

Multiplication and division prove each other. The quotient multiplied by the divisor produces the dividend, while the product divided by the multiplier gives the multiplicand.

Examples.

$$\begin{array}{r} 62{,}475{,}389 \\ 11 \\ \hline 11)\;687{,}229{,}279 \\ \hline 62{,}475{,}389 \end{array} \qquad \begin{array}{r} 11)\;764{,}891{,}849 \\ \hline 69{,}535{,}622\tfrac{7}{11} \\ 11 \\ \hline 764{,}891{,}849 \end{array}$$

Multiplication and division are proved as shown above. Add the remainder (if any) to the product to prove division.

Exercises.

1. Multiply 76,487,363 by 11. Divide 674,367,879 by 11.
2. " 39,367,462 by 11. " 763,986,479 by 11.
3. " 81,762,463 by 11. " 679,867,458 by 11.
4. " 97,863,987 by 11. " 967,489,653 by 11.
5. " 63,484,593 by 11. " 789,467,563 by 11.

30. Teach the use of 12 as a multiplier or a divisor.

Examples.

$$\begin{array}{r} 73{,}467{,}983 \\ 12 \\ \hline 12)\;881{,}615{,}796 \\ \hline 73{,}467{,}983 \end{array} \qquad \begin{array}{r} 12)\;876{,}492{,}571 \\ \hline 73{,}041{,}047\tfrac{7}{12} \\ 12 \\ \hline 876{,}492{,}571 \end{array}$$

Exercises.

1. Multiply 91,467,586 by 12. Divide 708,674,769 by 12.
2. Multiply 83,483,597 by 12. Divide 971,698,981 by 12.
3. Multiply 53,789,987 by 12. Divide 882,831,764 by 12.
4. Multiply 67,469,782 by 12. Divide 981,692,965 by 12.
5. Multiply 59,963,892 by 12. Divide 763,479,842 by 12.

31. Teach to multiply or divide by 11 or 12.

Exercises.

Divide 764,924,976 by 11 and multiply the quotient by 12.
Multiply 78,491,563 by 11 and divide the product by 12.
" 93,469,764 by 12 " " " " " 11.
Divide 876,468,501 by 12 " multiply " quotient " 11.
" 963,483,791 by 11 " " " " " 12.

Problems.

1. How much will 489 dozen eggs cost at 11 cents a dozen? at 12 cents a dozen?

2. How many slates, at 11 cents a piece, can be bought for $79,487.10? for $98,524.91?

NOTE.— The divisor and dividend are like numbers; the quotient will be times or slates.

3. If melons are worth 12 cents each, how many can be bought for $73,563.48? for $87,462.84?

4. Since there are 12 inches in one foot, how many feet are there in 962,474,436 inches?

5. Since 9 square feet make one square yard, how many square yards are there in 76,148,748 square feet?

6. Since 12 ounces make a pound Troy, how many ounces in a bar of silver weighing $59\frac{7}{12}$ lb.?

7. How many pounds of gold in 764,880 ounces?

8. If a man works 313 days in a year, 8 hours a day, at $7 an hour, how much will he receive for his labor in 9 years?

32. Teach how to multiply by a number containing two figures.

$$\begin{array}{r} 9{,}465{,}879 \\ 87 \\ \hline 66{,}261{,}153 \\ 757{,}270{,}32 \\ \hline 823{,}531{,}473 \end{array}$$

Explanation.—87 contains 7 units and 8 tens. Multiplying by the 7 units, the **partial** product 66,261,153 units is obtained. Multiplying by 8 tens, the **partial** product, 75,727,032 tens is obtained. By writing tens under tens and so on, and adding the partial products, the **complete** product is obtained.

Exercises.

1. Multiply 7,468,592 by 39, by 47, by 53, by 67.
2. Multiply 9,671,487 by 29, by 31, by 59, by 73.
3. Multiply 8,457,648 by 43, by 61, by 79, by 83.
4. Multiply 7,608,973 by 49, by 68, by 85, by 93.
5. Multiply 5,987,436 by 58, by 69, by 87, by 79.

Problems.

1. How much will 879 acres of land cost at $37 an acre?

2. At an average price of $92 a head, how much will 379 horses cost?

3. At $5.90 a barrel, how much will 97 barrels of flour cost?

4. How far will a horse go in 89 days, if he travels at the rate of 57 miles a day?

5. If 36 inches make a yard, how many inches are there in 5487 yards?

6. Since 32 quarts make a bushel, how many quarts are there in 978 bushels?

33. Teach how to divide by a number containing two figures:

```
8)998952(124869
  8       100000
 --        20000
  19        4000
  16         800
  --          60
   38  55      9
   32  48
   --  --
   69  72
   64  72
   --  --
```

Analysis.—Use the same form of analysis that is used in the example below. Show pupils that the form of long division is the same as that of short division with the exception that in the latter many processes are carried on mentally.

```
21)468972(22332
   42
   --
    48
    42
    --
     69
     63
     --
      67
      63
      --
       42
       42
       --
```

Analysis:—21 is contained in 46 ten-thousands, 2 ten-thousands' times and 4 ten-thousands remaining; annex the 4 ten-thousands to the 8 thousands brought down, making 48 thousands. 21 is contained in 48 thousands 2 thousands' times, and 6 thousands remaining. Annex the 6 thousands to the 9 hundreds brought down, making 69 hundreds. 21 is contained in 69 hundreds, 3 hundreds' times, and 6 hundreds remaining; annex the 6 hundreds to the 7 tens brought down, making 67 tens. 21 is contained in 67 tens 3 tens' times, and 4 tens remaining. Annex the 4 tens to the 2 units brought down, making 42 units. 21 is contained in 42 units 2 units' times and no units remaining. Write the sum of these quotients as the entire quotient.

Let the pupils analyze in the same manner:

468,982 ÷ 21; 65,785 ÷ 31; 86,965 ÷ 41; 1,496,967 ÷ 41; 38,379,994 ÷ 43; 182,345 ÷ 62; 789, 645 ÷ 94; 1,678.964 ÷ 85; 468,975 ÷ 78; 646,342 ÷ 49.

Rule for Long Division.

1. Write the divisor on the left of the dividend with a curved line between them.

2. Find how many times the divisor is contained in the fewest left hand orders of the dividend and write that number in the quotient.

3. Multiply the divisor by the number thus found and subtract its product from the orders divided.

4. To the remainder bring down the next figure of the dividend and proceed as in 2.

5. Write the final remainder over the divisor at the right of the quotient.

Caution 1.—When any product of the divisor by the quotient figure is greater than the orders from which it is to be subtracted, a smaller quotient figure must be used.

2. When any remainder is greater than the divisor a larger quotient figure must be used.

3. When any remainder with another figure brought down is less than the divisor, place a (0) in the quotient and bring down another figure from the dividend.

Proof.

Multiply the quotient by the divisor, adding the remainder to the product, which should then equal the dividend.

Exercises.

1. Divide 76,483,461 by 19, by 23, by 29, by 31.
2. Divide 59,375,892 by 37, by 41, by 47, by 53.
3. Divide 83,963,478 by 59, by 61, by 67, by 71.
4. Divide 91,487,982 by 73, by 79, by 83, by 89.
5. Divide 80,080,467 by 91, by 87, by 78, by 99.

Problems.

1. How many bushels of wheat at 83 cents a bushel can be bought for $787.67?

2. What is the price of wheat, if 949 bushels cost $787.67?

3. How fast does a train move, if it goes 1127 miles in 49 hours?

4. If a boat moves 97 miles in a day, how long will it take to go 2813 miles?

5. How many hhd. of 63 gallons will be required to hold 6930 gallons of molasses?

6. How many yards in 19360 inches?

7. How many bushels of beans in 3104 quarts?

8. If 56 lb. equals a bushel, how many bushels are there in 12,880 lb.?

9. Since a bushel of coal weighs 80 lb., how many bushels in a car load weighing 20080 lb.?

10. If in the excavation of a cellar 10 ft. deep, 45 ft. wide and 60 ft. long 27,000 cubic feet of dirt is removed, how many cart-loads of 27 cubic feet each will this amount of dirt make?

11. A drover spent $7,685 for cattle at an average price of $29 a head; he sold 187 head at $37 each and the remainder at $35 a head; how much did he gain?

12. How many gallons of molasses at 35 cents a gallon will pay for 6,745 pounds of butter at 27 cents a pound?

13. How many yards in 786,942 inches?

14. How many acres will $1,820 buy at the rate of $13 per acre?

15. A man received an annual salary of $2,500, and paid $175 for taxes, $117 as donations, $375 as rent, $349 for clothing, and $873 for household expenses; how much does he save annually?

Statement.

Received as salary		$2,500
Paid for taxes	$175	
“ “ donations	117	
“ “ rent	375	
“ “ clothing	349	
“ “ household expenses	873	1,889
Amount saved annually		$611

16. A lady bought a house and lot for $10,000, paying $2,938 cash. She afterwards paid, at one time, $1,973, and, at another time, $2,197; how much does she still owe?

☞ Make a statement as above.

17. An ice dealer bought 1,000 tons (2,000,000 pounds) of ice; he sold in June, 467,515 pounds; in July, 763,475 pounds; in August, 674,260 pounds; if he sold what remained in September at 65 cents per hundred pounds, how much did he receive for it?

18. A general had an army of 40,000 men and fought three battles; in the first battle he lost 7,342 men; in the second, 5,679 men; if he had only 21,237 men left after fighting the third battle, how many men did he lose in that battle?

19. A man traded a drove of 98 horses worth $175 each for a farm of 329 acres valued at $78 an acre; how much does he still owe?

20. A merchant gave his note for $5,200. He paid at one time $2,500 and at another time $176; how much does he still owe?

21. A farmer raised 1,575 bushels of wheat and 900 bushels of corn. He sold 675 bushels of wheat and 387 bushels of corn to A, and what remained of each to B; if B paid 85 cents a bushel for the wheat and 47 cents a bushel for the corn; how much did B pay the farmer?

22. A had 475 sheep and B 179 more than A; C has 211 sheep less than A and B; how many sheep has C?

Review and Home Exercises.

I.

1. Write from dictation 706,489; 90,700,642; 907; 679,000,089; 9,071; 714,000,608; 479; 87; 490,006,007; 543,041,000; 673,897; 90,008.

2. Add 907 + 892 + 673 + 946 +805 + 793 + 986 + 547 +862 +793 + 847 + 681 + 908.

3. From 901,867,461 take 868,463.

4. Multiply 987,465,687 by 98.

5. Divide 674,008,923 by 89.

II.

1. From 891,462,893 take 473,684.

2. Multiply 967,489,678 by 11.

3. Divide 673,487,009 by 12.

4. Multiply 67,897,387 by 79.

5. Divide 600,000,790 by 97.

III.

1. Write from dictation 709,600,004; 709; 6,004,047; 19,008,704; 674; 89; 717; 64; 8,940,604; 79,800,406; 789; 40; 67,084.

2. From 910,647,802 take 657,943.

3. Multiply 764,987,463 by 12.

4. Divide 742,362,981 by 12.

5. Divide 900,700,847 by 87.

IV.

1. Multiply 98,764,375 by 11.

2. From 701,467,804 take 477,915.

3. Divide 172,938,107 by 95.

4. Multiply 879,467,987 by 78.

5. Divide 904,829,183 by 86.

V.

1. Add 624; 575; 342; 391, 104; 1003.

2. Add 879 + 763 + 982 + 547 + 489 + 742 + 349 + 987 + 678 + 941 + 842 + 647 + 983.

3. Take 708,149 from 673,487,628.

4. Multiply 9,857,463 by 11 and divide the product by 12.

5. Divide 86958472 by 12 and multiply the quotient by 11.

VI.

1. Express the following numbers in words: 7,067,204; 901,673,487; 70,600,487; 734,898,694,567; 974,803,462.

2. Multiply 3,675,402 by 92, and divide the product by 11.

3. Express in figures and add: 7 hundred twenty-five; 12 hundred twenty-six; 9 thousand eighty-seven.

4. Multiply 769487 by 987.

5. Divide 987654321 by 739.

FRACTIONS.

DEFINITIONS.

34. **Numbers** are expressions for things.

35. A **Unit** is one of the kind of thing expressed by a number.

36. There are **two** kinds of things which may be expressed by numbers, **whole things** and **equal parts of things.**

37. An **Integer** is a number which expresses **whole** things.

38. A **Fraction** is an expression for one or more of the **equal parts** of a unit.

39. **Integers** are **written** by means of single numbers.

40. **Fractions** are **written** by means of two numbers, one of which is placed above a short horizontal line and the other below it.

41. The number placed below the short horizontal line is called the **Denominator.** It shows the **number of equal parts** into which the unit is divided.

42. The number placed above the short horizental line is called the **Numerator.** It shows how many of the equal parts of the unit are taken.

Exercises.

1. Divide any thing into two equal parts. What is one part called? (**One-half.**) How many halves in a whole thing? How many halves in three whole things?

2. Divide any thing into three equal parts. What is one part called? (**One-third.**) How many thirds in a whole thing? How many thirds in five whole things?

3. Divide any thing into four equal parts. What is one part called? (**One-fourth.**) How many fourths in a whole thing.? How many fourths in six whole things?

Suggestion.—Take long narrow strips of paper and let the pupil fold and tear them into any required number of equal parts. If torn into four equal parts, three parts is *three-fourths* of the *whole strip* of paper.

A fraction is always a **fraction of something.** Thus: one-half is one-half of one, or of some definite thing. Two-thirds is two of the three equal parts of some whole thing. **Realize fractional numbers before using figures to express them.**

4. Express the following fractions in figures: Three-fourths; four-sevenths; three-fifths; eight-ninths, eleven-twelfths; two-elevenths; seven-tenths; nine-twelfths; six-elevenths; four-ninths.

5. Analyze the fractions so written. Thus: $\frac{3}{4}$ is a fraction, because it is an expression for three of the four equal parts of a unit. 4 is the denominator, and shows the number of parts into which the unit is divided. 3 is the numerator, and shows how many of these parts are taken. $\frac{1}{4}$ expresses the fractional unit.

☞ $\frac{1}{4}$ means $\frac{1}{4}$ of 1; $\frac{3}{4}$ means $\frac{3}{4}$ of 1 or $\frac{1}{4}$ of 3.

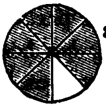
43. There are **two** kinds of fractions, **Proper Fractions** and **Improper Fractions.**

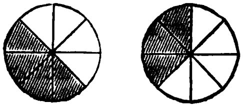
44. A **Proper Fraction** is a fraction whose numerator is **less** than its denominator. Thus: $\frac{3}{8}$ is a proper fraction, because its numerator is less than its denominator.

45. An **Improper Fraction** is a fraction whose numeraotr is **equal to** or **greater** than its denominator. Thus: $\frac{5}{4}$ is an improper fraction, because its numerator is greater than its denominator.

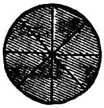
Illustrations.

By the fraction $\frac{7}{8}$ is meant, that one or more equal units have each been divided into 8 equal parts and that 7 of these parts are taken.

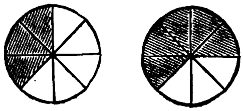
Thus:  a circle may be divided into 8 equal parts and 7 of the parts may be taken. Or two or more equal circles may be divided each into 8 equal parts and 7 of the parts may be taken. Thus:



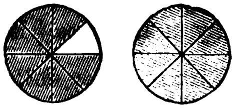
2. By the fraction $\frac{8}{8}$ is meant that one or more units have been each divided into 8 equal parts and that 8 of them have been taken. Thus:



or,



3. By the fraction $\frac{15}{8}$ is meant that two or more equal units have been each divided into 8 equal parts and that 15 of these parts are taken. Thus:



It will be seen therefore that a fraction may represent some of the equal parts of one thing, or some of the equal parts of several equal things.

46. A **Mixed Number** is an integer and a fraction united. Thus: $5\frac{3}{4}$ is a mixed number.

Exercises.

Analyze the following fractions and tell whether they are proper or improper:

$\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, $\frac{8}{4}$, $\frac{9}{4}$, $\frac{5}{6}$, $\frac{11}{6}$, $\frac{17}{6}$, $\frac{29}{6}$.

Model of Analysis.—$\frac{7}{4}$ is a fraction, since it is written by means of two numbers and notes equal parts of things. The denominator 4 shows that each thing has been divided into 4 equal parts. The numerator 7 shows that 7 of these parts are taken. $\frac{1}{4}$ expresses the fractional unit. The numerator is greater than the denominator; therefore $\frac{7}{4}$ is an improper fraction.

Improper fractions may be resolved into **whole numbers** or into **mixed numbers.** When the numerator is a multiple of the denominator, the value of the fraction is expressed by an integer. Thus: $\frac{3}{3} = 1$, $\frac{6}{3} = 2$, $\frac{9}{3} = 3$, $\frac{12}{4} = 3$; $\frac{15}{5} = 3$; $\frac{18}{6} = 3$; $\frac{24}{8} = 3$.

Change the following improper fractions into whole or mixed numbers.

$\frac{5}{4}$, $\frac{11}{5}$, $\frac{21}{7}$, $\frac{13}{6}$, $\frac{36}{9}$, $\frac{29}{11}$, $\frac{36}{12}$, $\frac{25}{10}$, $\frac{24}{8}$, $\frac{18}{8}$.

Model of Analysis.—Since 1 equals $\frac{4}{4}$, there are as many units in $\frac{5}{4}$ as $\frac{4}{4}$ is contained times in $\frac{5}{4}$, which is $1\frac{1}{4}$.

Change the following integers to improper fractions:

5 to thirds, 7 to fourths, 8 to fifths, 9 to sixths, 10 to sevenths, 8 to ninths, 11 to tenths, 12 to elevenths, and 9 to twelfths.

Model.—Since $1 = \frac{3}{3}$, $5 = 5$ times $\frac{3}{3}$,or $\frac{15}{3}$.

Change the following mixed numbers to improper fractions.

$2\frac{3}{4}$, $6\frac{4}{5}$, $7\frac{5}{6}$, $8\frac{9}{10}$, $9\frac{7}{11}$, $10\frac{4}{7}$, $9\frac{5}{8}$, $8\frac{9}{11}$, $9\frac{7}{8}$, $6\frac{2}{3}$.

Model of Analysis.—Since in 1 there are $\frac{4}{4}$, in $2\frac{3}{4}$ there are twice $\frac{4}{4}$ plus $\frac{3}{4}$, or $\frac{11}{4}$.

Change the following fractions to lowest terms by inspection :

$\frac{7}{14}$, $\frac{9}{27}$, $\frac{8}{32}$, $\frac{35}{40}$, $\frac{42}{48}$, $\frac{35}{49}$, $\frac{49}{56}$, $\frac{54}{63}$, $\frac{40}{50}$, $\frac{33}{77}$, $\frac{88}{132}$

ADDITION OF FRACTIONS.

47. **Principle 1.** If their fractional units are **alike**, given fractions may be added or subtracted by finding the sum or difference of their numerators. Thus : $\frac{3}{4} + \frac{7}{4} + \frac{11}{4} = \frac{21}{4} = 5\frac{1}{4}$; $\frac{13}{9} - \frac{7}{9} = \frac{6}{9} = \frac{2}{3}$.

48. **Principle 2.** If their fractional units are **unlike**, given fractions cannot be added or subtracted without being so subdivided as to be made alike.

The process of changing fractions from unlike to like units is called **the reduction of fractions to equivalent fractions having like denominators.**

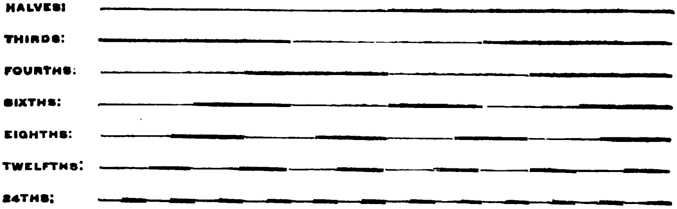
Suggestions.—Take three pieces of crayon. Divide one into **halves**, another into **thirds**, and the remaining piece into **fourths**. Take $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$ and ask, How much? There are six unlike pieces. If the $\frac{1}{2}$ be subdivided, it will equal $\frac{2}{4}$ and may be added to the $\frac{3}{4}$. But thirds and fourths cannot be added. If a third be divided into four equal parts and a fourth into three equal parts, the parts will be alike and can be added or subtracted. Each part will be $\frac{1}{12}$ of an entire piece of chalk, and the $\frac{1}{2}$ will equal $\frac{6}{12}$, the $\frac{2}{3}$ will equal $\frac{8}{12}$, and the $\frac{3}{4}$ will equal $\frac{9}{12}$, which may be added by adding their numerators. Thus:

$\frac{1}{2} = \frac{6}{12}$
$\frac{2}{3} = \frac{8}{12}$
$\frac{3}{4} = \frac{9}{12}$
—
$\frac{23}{12} = 1\frac{11}{12}$

Model of Analysis.—Since in 1 there are $\frac{12}{12}$, in $\frac{1}{2}$ there are $\frac{1}{2}$ of $\frac{12}{12}$, or $\frac{6}{12}$. In $\frac{1}{3}$ there are $\frac{4}{12}$ and in $\frac{2}{3}$ there are twice $\frac{4}{12}$, or $\frac{8}{12}$. In $\frac{1}{4}$ there are $\frac{3}{12}$, and in $\frac{3}{4}$ there are three times $\frac{3}{12}$, or $\frac{9}{12}$. The sum of $\frac{6}{12}$, $\frac{8}{12}$ and $\frac{9}{12}$ is $\frac{23}{12}$, or $1\frac{11}{12}$.

49. Rule for finding the common denominator by inspection.

If the largest denominator does not contain all the other denominators, try twice, three times or four times this number until a number is found which contains all the denominators.



The above scale will enable the pupil to see the equivalents of value in halves, thirds, fourths, sixths, eighths, twelfths and twenty-fourths. Other scales may be drawn to show other fractional units.

50. Find the common denominator of the following fractions and add them:

Example 1.— Add $\frac{3}{2}+\frac{4}{3}+\frac{3}{4}+\frac{5}{6}+\frac{11}{12}$.

$\frac{3}{2}$ $\frac{18}{12}$
$\frac{4}{3}$ $\frac{16}{12}$
$\frac{3}{4}$ $\frac{9}{12}$
$\frac{5}{6}$ $\frac{10}{12}$
$\frac{11}{12}$ $\frac{11}{12}$

$5\frac{1}{3}$ $\frac{64}{12}=5\frac{1}{3}$ Ans.

ANALYSIS.— Since 2, 3, 4 and 6 are exact divisors of 12, 12 is a common denominator of the fractions.

Rule.—Reduce the fractions to twelfths, add the numerators and reduce to a mixed number.

1. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{5}{12}$.
2. $\frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{8}{9}, \frac{11}{12}$.
3. $\frac{2}{3}, \frac{7}{9}, \frac{6}{7}, \frac{5}{6}, \frac{1}{2}$.
4. $\frac{5}{6}, \frac{3}{4}, \frac{7}{11}, \frac{5}{12}, \frac{1}{2}$.
5. $\frac{11}{12}, \frac{4}{9}, \frac{7}{8}, \frac{5}{6}, \frac{3}{4}$.
6. $\frac{1}{2}, \frac{2}{5}, \frac{3}{4}, \frac{9}{10}, \frac{7}{12}$.
7. $\frac{5}{6}, \frac{8}{9}, \frac{7}{8}, \frac{3}{4}, \frac{2}{3}$.
8. $\frac{4}{5}, \frac{3}{4}, \frac{5}{7}, \frac{9}{10}, \frac{3}{5}$.
9. $\frac{6}{7}, \frac{8}{9}, \frac{2}{3}, \frac{3}{4}, \frac{1}{2}$.
10. $\frac{5}{8}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{12}$.

51. Addition of mixed numbers.

Example 2.— Add $4\frac{3}{4}+7\frac{5}{8}+9\frac{11}{12}+8\frac{2}{3}+9$.

$4\frac{3}{4}$ $\frac{18}{24}$
$7\frac{5}{8}$ $\frac{15}{24}$
$9\frac{11}{12}$ $\frac{22}{24}$
$8\frac{2}{3}$ $\frac{16}{24}$
9

$39\frac{23}{24}$ $\frac{71}{24}=2\frac{23}{24}$ Ans.

Rule.—Reduce the fractional parts to fractions having a common denominator, add the numerators and reduce to a mixed number, and then add it to the integers.

52. Find the sum of the following fractional numbers:

1. $\frac{5}{6}+\frac{7}{8}+\frac{11}{12}+\frac{4}{9}+\frac{3}{4}$.
2. $7\frac{3}{4}+6\frac{5}{6}+4\frac{1}{2}+9\frac{11}{12}+7$.
3. $18\frac{6}{7}+15\frac{2}{3}+29\frac{4}{5}+36\frac{3}{7}+208$.
4. $209\frac{1}{2}+76\frac{2}{3}+378\frac{4}{5}+47\frac{9}{10}+98$.

5. $306\frac{3}{4}+81\frac{5}{6}+315\frac{4}{9}+89\frac{7}{12}+738$.

6. $1207\frac{3}{4}+7241\frac{3}{5}+93\frac{7}{8}+5005\frac{7}{10}+7801$.

7. $1708\frac{1}{2}+478\frac{3}{4}+1472\frac{5}{6}+903\frac{7}{9}+9207$.

8. $1371\frac{2}{3}+87\frac{5}{6}+12047\frac{8}{9}+904\frac{7}{12}+290401$.

9. $6004\frac{4}{5}+90091+1445\frac{7}{8}+2078\frac{9}{10}+7064\frac{7}{12}$.

10. $798\frac{3}{4}+9674\frac{8}{9}+7046\frac{7}{8}+7178\frac{11}{12}+76401$.

Problems.

1. A man bought four articles paying $\$\frac{2}{3}$, $\$\frac{3}{4}$, $\$\frac{7}{8}$ and $\$\frac{11}{12}$ for them respectively. How much did he pay for all?

2. A merchant bought five sacks of coffee, weighing $\frac{3}{2}$ cwt., $\frac{5}{4}$ cwt., $\frac{4}{5}$ cwt., $\frac{7}{8}$ cwt., and $\frac{9}{10}$ cwt. respectively. If a cwt. = 100 lb., how may pounds of coffee did he buy? [c = 100, wt. = weight.]

3. A merchant tailor has five remnants of cloth measuring $\frac{5}{8}$ yd., $\frac{3}{4}$ yd., $\frac{5}{6}$ yd., $\frac{8}{9}$ yd. and $\frac{7}{12}$ yd. How many yds. has he in all?

4. What is the aggregate length of five sticks measuring $6\frac{3}{4}$ ft., $5\frac{2}{3}$ ft., $8\frac{5}{6}$ ft., $4\frac{7}{8}$ ft. and $7\frac{11}{12}$ ft.?

5. A huckster has five barrels of potatoes containing respectively $2\frac{1}{2}$ bu., $1\frac{2}{3}$ bu., $2\frac{3}{4}$ bu., $2\frac{5}{6}$ bu. and $1\frac{11}{12}$ bu.? How many bushels are in the five barrels?

6. A tailor made five suits of clothes from a piece of cloth using $3\frac{1}{2}$ yd., $3\frac{3}{4}$ yd., $3\frac{5}{6}$ yd., $5\frac{1}{3}$ yd. and $4\frac{6}{7}$ yd. respectively. How much did he use in all?

7. A woman spent the following sums while shopping: $\$2\frac{2}{3}$, $\$4\frac{1}{4}$, $\$4\frac{7}{12}$ and $\$5\frac{5}{6}$. How much did she spend?

8. Farmer Brown owns $209\frac{7}{12}$ acres of land which is $98\frac{5}{12}$ acres less than Farmer Jones owns; how many acres does Farmer Jones own?

9. For how much must a horse that cost $\$279\frac{5}{6}$ be sold so as to gain $\$49\frac{8}{9}$?

10. A sold a house and lot to B for $\$4,378\frac{7}{12}$ and thereby lost $\$789\frac{9}{11}$; how much had A paid for the house and lot?

SUBTRACTION OF FRACTIONS.

53. Subtract a fraction from a fraction.

$$\begin{array}{ll} \frac{7}{8} & \frac{77}{88} \\ \frac{9}{11} & \frac{72}{88} \\ \hline \frac{5}{88} & \frac{5}{88} \text{ Ans.} \end{array}$$

Rule.— Reduce the fractions to a common denominator, and then find the difference between their numerators.

Exercises.

1. $\frac{3}{4}-\frac{2}{3}$. 3. $\frac{2}{3}-\frac{3}{5}$. 5. $\frac{9}{11}-\frac{3}{7}$. 7. $\frac{8}{11}-\frac{1}{4}$. 9. $\frac{7}{9}-\frac{2}{3}$.
2. $\frac{7}{8}-\frac{8}{11}$. 4. $\frac{7}{9}-\frac{5}{12}$. 6. $\frac{8}{9}-\frac{7}{12}$. 8. $\frac{7}{12}-\frac{3}{8}$. 10. $\frac{3}{5}-\frac{7}{12}$.

54. Subtract a whole number from a mixed number.

$$\begin{array}{r} 1,072,517,293\frac{5}{7} \\ 82,526,489 \\ \hline 989,990,804\frac{5}{7} \end{array}$$

Explanation.— When the fraction is attached to the minuend, it is simply brought down.

55. Subtract a mixed number from a whole number.

$$\begin{array}{r} 1,071,821,918 \\ 72,943,849\frac{7}{11} \\ \hline 998,878,068\frac{4}{11} \end{array}$$

Explanation.— When the fraction is attached to the subtrahend, it must be subtracted from 1 reduced to the same denomination.

Exercises.

1. From 8104321 take $98765\frac{7}{8}$.
2. From $7015364\frac{5}{11}$ take 764132.
3. From 83014768 take $67435\frac{8}{9}$.
4. From 9016794 take $774634\frac{7}{12}$.
5. From $70164298\frac{8}{11}$ take 746537.

56. Subtract a mixed number from a mixed number.

$$\begin{array}{r} 6{,}017{,}273\frac{5}{9} \\ 8{,}384\frac{7}{8} \\ \hline 6{,}008{,}888\frac{49}{72} \end{array} \qquad \begin{array}{r} \frac{40}{72} \\ \hline 112 \\ 63 \\ \hline 49 \end{array}$$

Rule.—When the numerator of the minuend is less than that of the subtrahend, add one reduced to the fractional form required. Thus $\frac{40}{72}$ is less than $\frac{63}{72}$, hence add $\frac{72}{72}$ to the minuend.

Exercises.

1. From $7012796\frac{5}{9}$ take $72476\frac{8}{11}$.
2. From $9102687\frac{7}{12}$ take $410289\frac{7}{9}$.
3. From $800176425\frac{5}{8}$ take $31468\frac{7}{9}$.
4. From $910026455\frac{5}{6}$ take $71845\frac{8}{11}$.
5. From $7041681\frac{5}{12}$ take $41687\frac{9}{11}$.

Problems.

1. A boy had $\$\frac{5}{6}$ and spent $\$\frac{2}{3}$ for a sled; how much had he left?

2. If a girl has $5 and spends $\$2\frac{8}{9}$ for a pair of shoes; how much has she left?

3. If a man should buy $9\frac{3}{4}$ bushels of apples and then sell $7\frac{5}{6}$ bushels; how much would he have left?

4. There are $17\frac{5}{9}$ bushels of apples in one bin and $14\frac{7}{11}$ bushels in another. If $20\frac{5}{6}$ bushels are sold, how many bushels are left?

5. If a man buy a pig for $\$4\frac{7}{11}$ and a sheep for $\$5\frac{6}{7}$, and sell them for $\$12\frac{5}{7}$, how much does he gain?

6. A farmer bought a cow for $\$27\frac{7}{11}$ and a calf for $\$5\frac{5}{8}$, and then sold them so as to gain $\$7\frac{5}{11}$; for how much did he sell them?

7. A drover sold a cow for $\$31\frac{5}{6}$ and thereby gained $\$4\frac{5}{9}$; how much did the cow cost?

8. If $\$3\frac{5}{9}$ are lost by selling a cow for $\$37\frac{7}{12}$; how much did she cost?

9. If a colt is bought for $\$36\frac{6}{7}$ and is sold so as to gain $\$7\frac{4}{5}$; for how much is the colt sold?

10. How much will be gained if a horse is bought for $\$87\frac{6}{11}$ and a cow for $\$31\frac{5}{9}$ and both are sold for $\$131\frac{4}{9}$?

11. How much will be lost if a horse that cost $\$125\frac{5}{6}$ and a buggy which cost $\$175\frac{4}{9}$ be sold for $\$231\frac{5}{12}$?

12. A drover bought sheep at $\$3\frac{5}{9}$ each, hogs at $\$4\frac{7}{12}$ each, cows at $\$26\frac{5}{8}$ each, how much will be made by selling a hog, a cow and a sheep for $\$47\frac{5}{6}$?

13. If a horse cost $\$89\frac{5}{6}$ and a cow cost $\$23\frac{1}{12}$, how much will be lost by selling them for $\$108\frac{8}{9}$?

14. A's farm contains $763\frac{5}{9}$ acres, which is $79\frac{7}{12}$ acres more than B's farm contains; how many acres are in B's farm?

15. A man owes the grocer $\$19\frac{7}{8}$, the butcher $\$19\frac{11}{12}$, and the baker $\$9\frac{8}{9}$. If he has $50, how much will remain after paying these three bills?

MULTIPLICATION OF FRACTIONS.

57. **Principle.**—A fraction is multiplied by increasing the number of its parts, or by increasing the size of its parts. The number of parts is increased by *multiplying the numerator*, and the size of the parts is increased by *dividing the denominator.*

58. Multiply a fraction by a whole number.

Example 1.—Multiply $\frac{5}{11}$ by 9.

$\frac{5}{11} \times 9 = \frac{45}{11}$.

$\frac{45}{11} = 4\frac{1}{11}$. Ans.

Analysis.—Nine times five-elevenths equals $\frac{45}{11} = 4\frac{1}{11}$

Example 2.—Multiply $\frac{7}{12}$ by 4.

$\frac{7}{12} \times 4 = \frac{7}{3}$.

$\frac{7}{3} = 2\frac{1}{3}$. Ans.

Analysis.—Since the denominator is a multiple of 4, divide it by 4.

Rule.—Multiply the numerator by the whole number, or, when the denominator is a multiple of the whole number, divide by it.

Exercises.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$\frac{6}{11}$	$\frac{8}{9}$	$\frac{7}{8}$	$\frac{6}{7}$	$\frac{11}{12}$	$\frac{4}{5}$	$\frac{9}{11}$	$\frac{5}{6}$
9	7	4	3	6	5	5	8
—	—	—	—	—	—	—	—

59. Multiply a mixed number by a whole number.

Example.—Multiply $469\frac{8}{11}$ by 7.

$$\begin{array}{r} 469\frac{8}{11} \\ 7 \\ \hline 5\frac{1}{11} \\ 3283 \\ \hline 3288\frac{1}{11}.\ \text{Ans.} \end{array} \qquad \frac{8}{11}\times 7=\frac{56}{11}=5\frac{1}{11}$$

Analysis.—Multiply the fraction by the whole number. Then reduce to a mixed number and add it to the product of the integral part.

Exercises.

1. $87\frac{5}{6}\times 9=$ 3. $46\frac{3}{7}\times 8=$ 5. $69\frac{5}{9}\times 3=$ 7. $47\frac{1}{12}\times 5=$

2. $89\frac{8}{9}\times 8=$ 4. $97\frac{6}{10}\times 9=$ 6. $85\frac{7}{11}\times 6=$ 8. $49\frac{8}{9}\times 8=$

60. Multiplying a whole number by a fraction.

$$\begin{array}{r} 674 \\ \frac{6}{7} \\ \hline 7)4044 \\ \hline 577\frac{5}{7}.\ \text{Ans.} \end{array} \qquad \begin{array}{r} 7(674 \\ \hline 96\frac{2}{7} \\ 6 \\ \hline 577\frac{5}{7}.\text{Ans.} \end{array}$$

Analysis.—6 times $\frac{1}{7}$ of 674 equals 6 times 674, or 4044 divided by 7, or $577\frac{5}{7}$. Or $\frac{1}{7}$ of $674=96\frac{2}{7}$ and $\frac{6}{7}$ of $674=6$ times $96\frac{2}{7}$, or $577\frac{5}{7}$.

Exercises.

9. $87\times\frac{9}{11}=$ 11. $907\times\frac{7}{9}=$ 13. $864\times\frac{5}{7}=$ 15. $825\times\frac{7}{12}=$

10. $763\times\frac{7}{8}=$ 12. $971\times\frac{7}{10}=$ 14. $76481\times\frac{5}{6}=$

61. Multiplying a fraction by a fraction.

Example.—Multiply $\frac{7}{12}$ by $\frac{5}{11}$.

$\frac{7}{12}\times\frac{5}{12}=\frac{35}{132}$. Ans.

Analysis.—Since $\frac{1}{11}$ of $\frac{7}{12}=\frac{7}{132}$, $\frac{5}{11}$ of $\frac{7}{12}=5$ times $\frac{7}{132}$, or $\frac{35}{132}$.

Rule. — Multiply the numerators together for a new numerator and multiply the denominators together for a new denominator.

Exercises.

1. $\frac{7}{8}$ by $\frac{8}{9}$; $\frac{6}{7}$ by $\frac{9}{11}$; $\frac{8}{9}$ by $\frac{11}{12}$; $\frac{4}{5}$ by $\frac{7}{12}$.
2. $\frac{7}{8}$ by $\frac{9}{11}$; $\frac{5}{6}$ by $\frac{7}{12}$; $\frac{4}{5}$ by $\frac{9}{10}$; $\frac{5}{9}$ by $\frac{11}{12}$.

62. Multiplying a mixed number by a fraction or by a mixed number.

Example 1. =Multiply $11\frac{3}{4}$ by $\frac{5}{9}$.

$\frac{47}{4} \times \frac{5}{9} = \frac{235}{36}$

$\frac{235}{36} = 6\frac{19}{36}$, Ans.

Example 2.—Multiply $18\frac{5}{7}$ by $9\frac{7}{8}$

$\frac{181}{7} \times \frac{79}{8} = \frac{10349}{56}$

$\frac{10849}{57} = 184\frac{45}{56}$, Ans.

Rule.—Reduce the mixed numbers to improper fractions. Then multiply the numerators together for a new numerator and the denominators for a new denominator, and reduce the fraction to a mixed number.

Exercises.

1. Multiply $89\frac{7}{11}$ by $7\frac{5}{8}$; $8\frac{3}{4}$; $4\frac{7}{8}$.
2. Multiply $8\frac{8}{9}$ by $9\frac{6}{7}$; $6\frac{2}{3}$; $9\frac{3}{5}$.
3. Multiply $67\frac{5}{6}$ by $7\frac{5}{12}$; $7\frac{3}{4}$; $8\frac{4}{7}$.
4. Multiply $93\frac{7}{9}$ by $3\frac{9}{11}$; $6\frac{5}{6}$; $9\frac{2}{3}$.
5. Multiply $78\frac{4}{5}$ by $7\frac{11}{12}$; $4\frac{3}{7}$; $8\frac{5}{9}$.
6. Multiply $97\frac{5}{9}$ by $9\frac{7}{8}$; $8\frac{2}{5}$; $9\frac{7}{8}$.
7. Multiply $83\frac{11}{12}$ by $9\frac{5}{7}$; $6\frac{1}{7}$; $8\frac{4}{11}$.
8. Multiply $807\frac{6}{11}$ by $3\frac{5}{9}$; $7\frac{2}{3}$; $6\frac{4}{5}$.
9. Multiply $706\frac{7}{12}$ by $8\frac{4}{5}$; $9\frac{3}{5}$; $7\frac{5}{6}$.
10. Multiply $749\frac{7}{9}$ by $8\frac{6}{7}$; $8\frac{2}{3}$; $9\frac{3}{4}$.

Problems.

1. What will $\frac{5}{6}$ bushels of linseed cost at \$4.00 a bushel?

2. At \$$\frac{5}{6}$ a piece, what will 9 caps cost?

3. What must be paid for 13 books at \$$\frac{11}{12}$ each?

4. How much will 11 lemons cost at $\frac{7}{8}$ cents each?

5. At \$$\frac{9}{10}$ a bushel, what will 9 bushels of wheat cost?

6. What will $\frac{47}{48}$ acres of land cost at \$16.00 an acre?

7. If one cap costs \$$\frac{11}{24}$, how much will 8 caps cost?

8. At \$$\frac{31}{35}$ a piece, what will 11 brooms cost?

9. How much must be paid for 24 shovels at \$$\frac{19}{72}$ each?

10. If one melon costs $9\frac{7}{8}$ cents, what will 97 melons cost?

11. At \$$1\frac{5}{6}$ each, how much will 79 books cost?

12. What will 689 pounds of beef cost at $11\frac{5}{6}$ cents a pound?

13. If one hog costs \$$5\frac{7}{12}$, what will 487 hogs cost?

14. At \$$4\frac{7}{8}$ a barrel, how much will 973 barrels of flour cost?

15. What is the cost of 79 arithmetics at \$$\frac{5}{9}$ each, and 197 readers at \$$\frac{7}{8}$ each?

16. How much must be paid for 579 bushels of wheat at \$$\frac{8}{9}$ a bushel, and 893 bushels of oats at \$$\frac{7}{12}$ a bushel?

17. How much more will 7 barrels of flour cost at \$$4\frac{9}{11}$ each than three loads of hay at \$$7\frac{5}{12}$ a load?

18. How much change will I get, if I buy 7 yards of cloth at \$$2\frac{5}{12}$ a yard and give the clerk a twenty-dollar bill?

19. A lady bought 6 yards of lace at $\$\frac{2}{7}$ a yard, 8 yards of ribbon at $\$\frac{2}{11}$ a yard and 6 handkerchiefs at $\$\frac{3}{7}$ each. How much change should she receive from a ten-dollar bill?

20. A green grocer bought 9 melons at $\$\frac{2}{9}$ each, 3 barrels potatoes at $\$1\frac{5}{9}$ a barrel, and 11 pine apples at $12\frac{5}{6}$ cents each. If he gives the dealer two five-dollar bills, how much change should he receive?

21. What is the cost of 426 lemons at $\frac{5}{6}$ cents each?

22. How much will 879 bushels oats cost at $\$\frac{2}{11}$ a bushel?

23. At $\$\frac{2}{9}$ a foot, how much will 987 feet of walnut lumber cost?

24. How much should be paid for 569 sheets of paper at $\frac{3}{7}$ cents a sheet?

25. At $\$\frac{2}{11}$ a bushel, how much will 547 bushels of peaches cost?

26. How much more must be paid for 329 melons at $\$\frac{2}{9}$ each than for 761 oranges at $\$\frac{1}{12}$ each?

27. If I buy 11 apples at $\frac{2}{3}$ cents each and 9 lemons at $\frac{3}{5}$ cents each, how much change will I get from two dimes?

28. John can do $\frac{3}{11}$ of a piece of work in one day and James can do $\frac{2}{9}$ of it in one day. In 9 days, how much more work can John do than James?

29. How much will $\frac{7}{9}$ of a melon cost, if a whole melon costs $\$\frac{2}{9}$?

30. If one box of soap costs $\$\frac{2}{5}$, how much will $\frac{5}{6}$ of a box cost?

31. How much must be paid for $\frac{7}{11}$ pounds of tea at $\$\frac{5}{12}$ a pound?

32. At $\$\frac{7}{11}$ a pound, what is the cost of $\frac{7}{9}$ pound of cheese?

33. What is the cost of $\frac{6}{7}$ pound of coffee at $\$\frac{2}{11}$ a pound?

34. How much more must be paid for $\frac{8}{11}$ pound of tea at $\$\frac{5}{12}$ a pound than for $\frac{7}{12}$ pound of sugar at $\$\frac{1}{12}$ a pound?

35. A grocer sells $\frac{5}{6}$ pound of tea at $\$\frac{3}{8}$ a pound, $\frac{9}{11}$ pound of cheese at $\$\frac{2}{9}$ a pound, and $\frac{11}{12}$ pound of coffee at $\$\frac{2}{9}$ a pound. If he receive a five-dollar bill in payment, how much change must he give?

36. How much will $817\frac{5}{6}$ bushels of apples cost at $\$\frac{3}{11}$ a bushel?

37. What will $917\frac{7}{11}$ pounds of butter cost at $\$\frac{2}{7}$ a pound?

38. At $\$\frac{5}{12}$ a gallon, how much should be paid for $937\frac{5}{6}$ gallons of molasses?

39. If one barrel of flour costs $\$5\frac{3}{8}$, how much will $847\frac{6}{11}$ barrels cost?

40. How much will $589\frac{5}{8}$ pounds of mutton cost at $\$\frac{2}{7}$ a pound?

41. How much will $9\frac{5}{8}$ pounds of coffee cost at $11\frac{7}{9}$ cents a pound?

42. If a man can walk $3\frac{7}{8}$ miles in an hour, how far can he walk in $8\frac{5}{6}$ hours?

43. What will $8\frac{7}{8}$ bushels of clover seed cost at $\$6\frac{7}{12}$ a bushel?

44. How much should be paid for $27\frac{7}{8}$ pounds of beef at $9\frac{5}{7}$ cents a pound?

45. At $\$6\frac{7}{9}$ a cord, how much would $973\frac{11}{12}$ cords of wood cost?

46. Multiply $869\frac{7}{8}$ by $967\frac{8}{9}$.

DIVISION OF FRACTIONS.

63. Principle.—A fraction is divided by diminishing the number of its parts, or by diminishing the size of its parts.

The number of its parts are diminished by dividing the numerator; the size of the parts, by multiplying the denominator.

64. To divide a fraction by a whole number.

Rule.—Divide the numerator, if it is a multiple of the divisor; if not, multiply the denominator.

Examples.—Divide $\frac{7}{8}$ by 7; $\frac{9}{11}$ by 8; $\frac{7}{12}$ by 9.

$$\tfrac{7}{8} \div 7 = \tfrac{1}{8};\quad \tfrac{9}{11} \div 8 = \tfrac{9}{88};\quad \tfrac{7}{12} \div 9 = \tfrac{7}{108}.$$

Exercises.

1. $\frac{9}{11} \div 3 = ?$
2. $\frac{11}{7} \div 5 = ?$
3. $\frac{1}{4} \div 7 = ?$
4. $\frac{7}{9} \div 9 = ?$
6. $\frac{20}{23} \div 7 = ?$
6. $\frac{17}{23} \div 7 = ?$
7. $\frac{5}{9} \div 8 = ?$
8. $\frac{7}{12} \div 7 = ?$
9. $\frac{6}{11} \div 6 = ?$
10. $\frac{5}{9} \div 7 = ?$
11. $\frac{9}{12} \div 9 = ?$
12. $\frac{7}{12} \div 5 = ?$
13. $\frac{6}{7} \div 6 = ?$

65. To divide a mixed number by a whole number.

Example.—Divide $3\frac{4}{7}$ by 6; —$9\frac{6}{7}$ by 8.

1. $6)\underline{3\frac{4}{7} = \frac{25}{7}}$

$\frac{25}{42}$ Ans.

2. $8)\underline{9\frac{6}{7}}$ $\qquad 1\frac{6}{7} = \frac{13}{7}$

$1\frac{13}{56}$ Ans.

If the mixed number is less than the divisor reduce it to an improper fraction and divide as in 64; if the mixed number is greater than the divisor, divide the whole number first and then divide the remainder as in 64.

Exercises.

1. $5)\ 4\frac{3}{7}$ $7)\ 9\frac{5}{11}$ $9)\ 121\frac{5}{12}$ $8)\ 137\frac{7}{9}$

2. $7)\ 763\frac{5}{8}$ $9)\ 487\frac{8}{11}$ $11)\ 675\frac{8}{9}$ $12)\ 489\frac{7}{11}$

66. To Divide a whole number by a fraction.

Principle.—One number can measure another only when their units are alike, and hence whole numbers, or fractions should be reduced to like *fractional* units.

Example.—Divide 9 by $\frac{7}{11}$.

$\frac{7}{11})\ 9$

$\frac{7}{11})\ \frac{99}{11}$

$14\frac{1}{7}$.Ans.

Analysis.—Since $1 = \frac{11}{11}$, $9 = \frac{99}{11}$, and $\frac{7}{11}$ is contained $14\frac{1}{7}$ times in $\frac{99}{11}$.

Rule.—Reduce the divisor and dividend to like fractional units and divide the numerator of the dividend by the numerator of the divisor.

Exercises.

3. $\frac{8}{9})\ 37$ $\frac{7}{8})\ 63$ $\frac{5}{7})\ 59$ $\frac{9}{11})\ 73$ $\frac{7}{12})\ 87$

Whenever the divisor is a fractional number, reduce to a common denominator.

67. To divide a fraction by a fraction.

Fractions of this kind are usually called **complex fractions**, having a fraction for each term. All forms of multiplication of fractions may be written as **compound fractions**, and all forms of division of fractions may be written as **complex fractions.**

Examples.—Divide $\frac{8}{9}$ by $\frac{7}{11}$; — $\frac{11}{12}$ by $\frac{6}{7}$.

$$\begin{array}{rl} \frac{7}{11}) & \frac{8}{9} \\ & \overline{} \\ \frac{63}{99}) & \frac{88}{99} \\ & \overline{} \\ & \frac{88}{63} = 1\frac{25}{63}.\text{Ans.} \end{array} \qquad\qquad \begin{array}{rl} \frac{6}{7}) & \frac{11}{12} \\ & \overline{} \\ \frac{72}{84}) & \frac{77}{84} \\ & \overline{} \\ & \frac{77}{72} = 1\frac{5}{72}.\text{Ans.} \end{array}$$

☞ **Reduce the fractions to a common denominators, and then divide.**

Exercises.

1. $\frac{11}{12})\ \frac{8}{9}$; $\quad\frac{4}{9})\ \frac{7}{11}$; $\quad\frac{9}{11})\ \frac{7}{12}$; $\quad\frac{7}{12})\ \frac{5}{6}$; $\quad\frac{5}{9})\ \frac{6}{11}$;

2. $\dfrac{\frac{6}{7}}{4\frac{1}{2}}$; $\quad\dfrac{8\frac{3}{4}}{\frac{6}{7}}$; $\quad\dfrac{\frac{5}{6}\text{ of }\frac{7}{8}}{6\frac{5}{11}}$; $\quad\dfrac{8\frac{1}{2}+6\frac{2}{3}}{9\frac{3}{4}-4\frac{5}{6}}$.

68. To divide a mixed number by a fraction.

Rule.—Reduce the mixed number to an improper fraction, and then reduce the fractions to common denominators.

Example.—Divide $7\frac{8}{9}$ by $\frac{7}{12}$.

$$\begin{array}{rl} \frac{7}{12}) & 7\frac{8}{9} = \frac{71}{9} \\ & \overline{} \\ \frac{63}{108}) & \frac{852}{108} \\ & \overline{} \\ & \frac{852}{63} = 13\frac{11}{21}\text{ Ans.} \end{array} \qquad\qquad \begin{array}{rl} 63) & 852\,(13 \\ & \underline{63} \\ & 222 \\ & \underline{189} \\ & \frac{33}{63} = \frac{11}{21}. \end{array}$$

Exercises.

1. $\frac{8}{9})\ 13\frac{5}{6}$; $\frac{7}{11})\ 175\frac{6}{7}$; $\frac{7}{12})\ 841\frac{5}{6}$; $\frac{6}{7})\ 467\frac{5}{6}$.

2. $\frac{7}{9})\ 6273\frac{4}{5}$; $\frac{7}{8})\ 91046\frac{11}{12}$; $\frac{6}{11})\ 87462\frac{3}{4}$

69. To divide a mixed number by a mixed number.

Example.—Divide $18\frac{7}{11}$ by $7\frac{5}{6}$.

$$\begin{array}{rl}
7\frac{5}{6}) & 18\frac{7}{11} \\
 & \overline{} \\
7\frac{55}{66}) & 18\frac{42}{66} \\
66 & 66 \\
\overline{} & \overline{} \\
42 & 108 \\
42 & 108 \\
55 & 42 \\
\hline
517) & 1230\ (2\frac{196}{517}\ \text{Ans.} \\
 & 1034 \\
 & \overline{} \\
 & 196
\end{array}$$

Rule.—Reduce the fractional parts to a common denominator, and reduce the mixed numbers to improper fractions and divide one numerator by the other, as in the example.

Reduce the fractional divisor and dividend to the same fractional unit and then dividing one numerator by the other, is the true divisional process. **Inverting the terms** of the divisor and multiplying has the same effect, but it is too difficult to understand.

Exercises.

3. $9\frac{8}{9})\ 67\frac{7}{11}$; $8\frac{7}{12})\ 98\frac{7}{8}$; $11\frac{5}{6})\ 98\frac{5}{12}$.

4. $8\frac{5}{7})\ 69\frac{4}{5}$; $12\frac{5}{6})\ 79\frac{8}{11}$; $8)\ 86\frac{9}{11}$.

70. Fourth Grade Forms in Fractions.

$$\begin{array}{rl} 34\frac{8}{9} &= 34\frac{64}{72} \\ 27\frac{7}{8} &= 27\frac{63}{72} \\ 26\frac{5}{6} &= 16\frac{60}{72} \\ 19\frac{11}{12} &= 19\frac{66}{72}.\ \text{Ans.} \\ \hline & 109\frac{37}{72} \end{array} \qquad \begin{array}{r} 72)\,253\,(3 \\ 216 \\ \hline 37 \end{array} \qquad \begin{array}{rl} 17\frac{7}{10} &= 17\frac{42}{60} \\ 8\frac{11}{12} &= \ \ 8\frac{55}{60} \\ \hline & 8\frac{47}{60}.\ \text{Ans.} \end{array} \qquad \begin{array}{r} 60 \\ 42 \\ \hline 102 \end{array}$$

$\frac{7}{8} \times 9 = \frac{63}{8} = 7\frac{7}{8}$. Ans.

$$\begin{array}{r} 3678\frac{8}{9} \\ 7 \\ \hline 25752\frac{2}{9}.\ \text{Ans} \end{array}$$

Pupils should be required to give intermediate and final results orally.

$17 \times \frac{7}{8} = ?$

$$\begin{array}{l} 8)\underline{17} \\ \ \ \ 2\frac{1}{8} \\ \ \ \ 7 \\ \hline 14\frac{7}{8}.\ \text{Ans.} \end{array} \qquad \begin{array}{r} 375 \\ 27\frac{8}{9} \\ \hline 333\frac{1}{3} \\ 2625 \\ 750 \\ \hline 10458\frac{1}{3}.\ \text{Ans.} \end{array} \qquad 41\frac{2}{3}$$

$\frac{7}{8} \times \frac{8}{9} = \frac{56}{72} = \frac{7}{9}$. Ans.

$\frac{8}{11} \div 4 = \frac{2}{11}$ Ans. or, $\frac{9}{11} \div 4 = \frac{9}{44}$ Ans.

$$\begin{array}{l} 7)\,379\frac{8}{9} \\ \hline 54\frac{17}{63}.\ \text{Ans.} \end{array} \qquad 1\frac{8}{9} = \frac{17}{9}$$

$46 \div \frac{7}{8} = ?$

$$\begin{array}{r} 46 \\ 8 \\ 7)\overline{368} \\ \hline 52\frac{4}{7}.\ \text{Ans.} \end{array}$$

$16 \div 5\frac{7}{8} = \ ?$

$$\begin{array}{r} 5\frac{7}{8})16 \\ 8 \\ 47(\overline{128}(2\frac{34}{47}.\ \text{Ans.} \\ 94 \\ \hline 34 \end{array}$$

$\frac{11}{12} \div \frac{8}{9} = \frac{33}{36} \div \frac{32}{36} = 1\frac{1}{82}$. Ans.

Problems.

1. If 9 heads of cabbage cost $\$\frac{9}{117}$, how much does one cost? — 5? — 9? — 11?

2. How much will one orange cost, if 11 oranges cost $\$\frac{2}{7}$? —4? —7? —9? —12?

3. If 8 pounds of beef cost $\$\frac{8}{11}$, how much will five pounds cost? — 6? — 8? — 10? —12?

4. What will 9 pounds of ham cost if 7 pounds cost $\$1\frac{2}{9}$? — 5? — 8? — 11? — 4?

5. If 7 pounds of cheese cost $\$\frac{5}{6}$, what will 12 pounds cost? —9? —6? — 11? — 4?

6. If 9 melons cost $\$1\frac{5}{6}$, what will one melon cost? — 4? 7? — 9? — 11?

7. If 7 bushels of potatoes cost $\$4\frac{8}{9}$, what is the price per bushel?

8. What is the price of flour per barrel, if 12 barrels cost $\$69\frac{8}{11}$?

9. If $\$1\frac{1}{5}$ are paid for a turkey weighing 11 pounds what is the price per pound? How much will an 8-pound turkey cost?

10. If $\frac{7}{11}$ barrels flour costs \$6, how much is one barrel worth?

11. How much must be paid for 9 yards of silk velvet, if $\frac{7}{12}$ yards cost \$5?

12. If $\frac{7}{8}$ of a cord of wood cost \$9, what will one cord cost? — 7? — 9? — 11? — 4?

13. If 14 cents are paid for $\frac{5}{7}$ pounds of butter, what is the price of one pound?

14. If $\frac{7}{12}$ of a melon cost $\$\frac{2}{7}$, how much will one melon cost? — 7? —9? — 12? — 4?

15. If $\frac{3}{7}$ of a cord of wood costs $\$\frac{3}{5}$, how much is one cord worth? — 11?–8? — 5? — 10?

16. What is the price of butter per pound, if $\frac{3}{11}$ pounds costs $\$\frac{3}{5}$? — 8? —6? — 5? —11?

17. If $\$\frac{5}{6}$ be paid for $\frac{7}{12}$ barrels of flour, what is the price of one barrel?— 12?— 4?— 9?— 11?

18. A grocer paid $\$\frac{7}{11}$ for $\frac{5}{12}$ barrels apples, how much should he pay for 9 barrels? — 7? — 10? — 4? — 12?

19. A farmer sold $371\frac{5}{6}$ acres of land, which was $\frac{5}{6}$ of his farm; how many acres did his farm contain?

20. A tailor bought $23\frac{5}{12}$ yards of cloth, which was $\frac{7}{11}$ of the entire piece; how many yards did the piece contain?

21. A farmer has 470 sheep in one field, which is $\frac{5}{9}$ of all he has; how many sheep has he?

22. How many pupils are enrolled in a school, if there are 686 boys enrolled, and the boys are $\frac{7}{12}$ of the whole number?

23. If 7 barrels of flour cost $\$29\frac{3}{4}$, how much will $\frac{5}{6}$ barrels cost?—9?—7?—11?—4?

24. At $\$\frac{7}{8}$ a bushel, how much rye will $\$9\frac{5}{6}$ buy?

25. At $\$\frac{5}{8}$ a yard, how much cloth will $\$8\frac{7}{11}$ buy?

26. If B can walk $2\frac{7}{8}$ miles an hour, how long will it take

him to go and return from a town $14\frac{5}{8}$ miles distant, provided he remains $1\frac{7}{8}$ hours in the town?

27. A farmer has $\$43\frac{7}{8}$ to expend for clover seed at $\$3\frac{5}{8}$ a bushel; how many bushels can he buy?

28. The distance between two towns is $79\frac{8}{11}$ miles. If A can walk this distance in $19\frac{3}{4}$ hours and B in $21\frac{5}{6}$ hours; how far can each walk in one hour?

29. If C can travel $29\frac{3}{4}$ miles in one day and D can travel $37\frac{11}{12}$ miles, how long will it take C to travel as far as D goes in $14\frac{7}{8}$ days?

30. A tailor has $\$49\frac{6}{7}$ and gives $\frac{2}{7}$ of it to his son and $\frac{4}{5}$ of the remainder to his daughter. How many yards of cloth at $\$2\frac{7}{8}$ a yard can he buy with the balance?

31. The distance between St. Louis and St. Paul is 729 miles. If two boats should leave St. Louis and St. Paul at the same time, one going down the river at the rate of $8\frac{5}{6}$ miles an hour and the other going up at the rate of $7\frac{11}{12}$ miles an hour, in how many hours would they meet? How far would one boat be from St. Paul, when the other reaches St. Louis?

32. A butcher sells $71\frac{3}{4}$ pounds of ham at 16 cents a pound and $171\frac{3}{4}$ pounds of lard at 12 cents a pound. After he has paid for 314 pounds of sugar at $7\frac{1}{2}$ cents a pound and $18\frac{5}{8}$ pounds of coffee at 24 cents a pound, how much tea can he buy with the balance at $81\frac{2}{3}$ cents a pound?

33. A farmer buys $2\frac{3}{4}$ yards of cloth at $\$3\frac{7}{8}$ a yard; $14\frac{1}{4}$ yards of silk at $\$1\frac{5}{8}$ a yard, and a web of muslin containing $48\frac{7}{8}$ yards at $12\frac{1}{2}$ cents a yard, and sells $17\frac{7}{8}$ pounds of butter at $16\frac{2}{3}$ cents a pound and $8\frac{3}{4}$ bushels of potatoes at 50 cents

a bushel. How many bushels of corn at $47\frac{5}{8}$ cents a bushel will balance the bill?

34. John bought $18\frac{3}{8}$ pounds of sugar, which was $3\frac{3}{7}$ pounds more than Henry bought; how many pounds did he buy?

35. One farm contains $197\frac{1}{12}$ acres of land, which is $19\frac{3}{8}$ acres less than another farm contains; how much is the other farm worth at $\$37\frac{3}{8}$ an acre?

36. E bought 84 tons of coal, of which $\frac{2}{7}$ was delivered in January, $\frac{3}{11}$ in February. Now if E paid $\$6\frac{3}{8}$ a ton for the coal and sold what remained for $\$8\frac{5}{8}$ a ton, how much did he make on what he sold?

37. A wholesale grocer sold the following articles:

13397 lb. leaf lard at $6\frac{5}{8}$ cents per lb.; $3\frac{7}{8}$ bu. beans at $2 per bu.; $27\frac{8}{11}$ bu. flaxseed at $3.10 per bu.; $5\frac{3}{8}$ bu. clover seed at $8 per bu.; 109 bbls. flour at $\$5\frac{7}{12}$. What was the total amount of his sales?

38. A merchant gave his note for $7567.39. He paid at one time $897.63 and at another time $3763.79. How much is still due?

39. A traveler who was 1367 miles from home, traveled homeward one week 235 miles; the next week 367 miles, and the third week 482 miles. How far from home was he at the end of the third week?

40. A man deposited in bank $9674. He drew four checks for $4367, $879, $2364 and $2169 respectively; how much money has he still in bank?

41. A farmer paid $6987 for a tract of land. He afterwards paid $967 for fencing and $3789 for other improvements. If he sells his farm for $12654, how much does he gain?

DECIMALS.

Definitions.

71. The **unit of value** for the United States is the **dollar** ($), — the silver dollar, the gold dollar, and the one-dollar bill (= 100 cents.)

Our **money** consists of the **coin** and **bank-bills** which constitute the metallic and paper currency in common use.

Multiples—The money which is a multiple of the unit of value is the two-dollar bill, the quarter-eagle ($2.50 in gold), the three-dollar gold coin, the five dollar bill, the half eagle ($5.00 in gold) the ten-dollar bill, the eagle ($10.00 in gold), the twenty-dollar bill, the double-eagle ($20.00 in gold), the fifty-dollar bill, the hundred-dollar bill, the five-hundred-dollar bill, and the thousand-dollar bill.

Fractions. — The money which is a fraction of the unit of value is the half-dollar (50 cents in silver), quarter-dollar (25 cents in silver), dime (ten cents in silver), half-dime (five cents in silver), nickel (five cents in nickel), two-cent piece (copper), and one-cent (copper). These are called **subsidiary coins** and are of less intrinsic value than the dollar. Bank-bills of less value than a dollar constitute **fractional currency** and are useful for remitting small amounts.

72. **VALUES** are computed in dollars, cents and mills, but accounts are kept in dollars and cents only.

The cent is the hundredth part of a dollar and the mill is the thousandth part of a dollar.

73. Values conform to the decimal scale, increasing from right to left and decreasing from left to right. The dollar sign ($) is written to the left of dollars and a period (.), called the decimal point, is placed between the dollars and cents. Cents are always expressed by means of two figures, and are so many hundredths of a dollar. Thus: eight dollars and seven cents is written $8.07, or $\$8\frac{7}{100}$.

Exercises.

1. What is the sum of $407.73, $89.63, $3,483.62, $79.41, $8.73, $467.81, $95.19, $21,463.87, $4,016.73, $9.87, $27,082.87, $721.82, and $671.91 ?

Rule. — Write dollars under dollars.

2. A boy found a package of money in which were 3 thousand-dollar bills, 7 five-hundred-dollar bills, 4 hundred-dollar bills, 7 fifty-dollar bills, 9 twenty-dollars bills, 17 ten-dollar bills, 6 five-dollar bills, 9 two-dollar bills, 7 double-eagles, 11 eagles, 14 half-eagles, 17 quarter-eagles, 7 silver dollars, 11 gold dollars, 19 half-dollars, 23 quarter-dollars, 26 dimes, 29 nickles, 7 half-dimes, 17 two-cent pieces, and 16 cents; how much money was in the package ?

Suggestion. — 17 ten-dollar bills = $170.00
25 quarter-dollars = $6.25

74. A **Decimal** is a number written in integral form whose value is less than one. It may also be regarded as a fraction whose denominator is 10 or some power of 10 unexpressed.

Decimals have the **decimal point** (.) written to the left of them. Thus: 7 tenths is .7; 85 hundredths is .85; 469 thousandths, is .469: 2349 ten-thousandths is .2349; 47473 hundred-thousandths is .47473; 921467 millionths is .921467.

75. Rule for the Numeration of Decimals.

Begin at the figure next to the decimal point (.) and calling it tenths, proceed toward the right, saying in order, tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, until the last figure is reached, which names its denomination. The numerator is read as in whole numbers.

Exercises.

(1)	(2)
6,734.876	4,674.89
89.74763	68.7464
763.009467	9.00678
83.83004	73,047.070009
6,761.734989	638.8403
87.000068	71.000007
.00763	831,242.0427
854.5237	904.92006
9.80204	76.819
708.008009	9.72346

☞ Use AND only between whole numbers and decimals. Thus; 8.17 is read, eight AND seventeen hundredths.

Write the above numbers from dictation.

☞ Always place TENTHS under TENTHS, and so on.

ADDITION AND SUBTRACTION OF DECIMALS.

76. As the value of units increase from right to left and decrease from left to right, the addition and subtraction of decimals are subject to the same rules as whole numbers.

Exercises.

☞ Write from dictation.

1. Find the sum of 63.7402; 789.824; 67.0271; 6,754.82713; 5,934.721942; 79.8291; 6.47; 832.90645; 79.7329; 861.74817; 71.8046; 593.4715; 684,927.800305.

2. What is the difference between 731.09 and 69.07459?

3. Find the sum of 9.804062; 901.678; 89.7; 467.0801; 761.8047; 962.81; 93.47891; 631.4878; 73.90189; 7,634.812; 7,329.54201; 76,947.801.

4. What is the difference between the sum of 872.967+89.467+9,681.4709 and the sum of 6.9472+891.68974+462.901809.

5. What is the value of 67.809—59.731+892.763—97.84327 + 698.47586 — 987.46 + 891.49 — 769.3842 + 972.74368 + 7.089 + 7,924 — 69.87946?

Problems.

1. The cultivated farms in a certain township contain respectively 129.07 A., 89.91 A., 237.87 A., 49.73 A., 349.57 A., 87.9 A., 689.79 A., 491.49 A., 327.9 A., 49.8 A., 67.98 A., 83.93 A., and 547.69 A.; how much tilled land does the township contain?

2. A merchant's sales last year were $76,289.73, which is $9,891.89 more than this year's sales; what is the amount of this year's sales?

3. A house and lot were sold for $7,367.89, which was $789.91 more than they cost; what was the cost?

4. A house and lot were bought for $9,867.97 and sold so as to gain $897.22; for how much were they sold?

5. An article that cost $7.63 was sold at a loss of $1.97; for how much was it sold?

MULTIPLICATION OF DECIMALS.

Examples.—Multiply .9 by 7.; — .09 by .07; — .009 by .007.

$\frac{9}{10} \times \frac{7}{10} = \frac{63}{100} = .63$; or, .9 × .7 = 63 Ans.

$\frac{9}{100} \times \frac{7}{100} = \frac{63}{10000} = .0063$; or, .09 × .07 = .0063 Ans.

$\frac{9}{1000} \times \frac{7}{1000} = \frac{63}{1000000} = .000063$; or .009 × .007 = .000063 Ans.

Rule.—Multiply the integers as in whole numbers and point off as many decimal places as there are decimal places in both the multiplier and multiplicand. When there are fewer places in the product than is required, prefix ciphers to make the required number of places.

77. Tenths × tenths = hundredths; hundredths × hundredths = ten-thousandths; thousandths × thousandths = millionths.

Exercises.

1. Multiply 93.708 by .987; — by 90.708.
2. Multiply 765.097 by .89; — by .0078.
3. Multiply 97.098 by 76.27; — by .9089.
4. Multiply 67.468 by .678974.
5. Multiply .000009 by .000008.

Problems.

1. What is the cost of 78.59 tons of coal at $7.68 a ton?

When there are five mills or more, increase the cents by 1 in the product and disregard the remaining decimals.

2. How far will a commercial traveler go in a year, if he travels 79.78 miles a day during 312.56 days?

3. What is the value of nine bins of wheat each containing 98.64 bushels, at 79 cents a bushel?

4. A livery stable keeper bought 9 car loads of hay averaging 83.69 cwt. per car at 95 cents per cwt.; how much did the hay cost?

5. What is the cost of 719.87 acres of land at $29.87 an acre?

DIVISION OF DECIMALS.

Example.—Divide 4.65 by 1.5.

```
1.5)4.65(3.1
    45
    ——
     15
     15
     ——
```

Analysis.—Since the dividend is the product of the divisor and quotient and contains as many decimal places as the divisor and quotient together, the quotient must contain as many decimal places as the difference between the number of decimal places in the dividend and those of the divisor.

78. **Rule.**—Divide as in integers, annexing ciphers to the dividend if necessary, to make the decimal places equal to the number of decimal places in the divisor. Point off as many

decimal figures at the right of the quotient as the decimal figures of the dividend exceed those of the divisor.

If the number of figures in the quotient is less than the number to be pointed off, prefix ciphers to make them the necessary number.

Exercises.

Divide:

1. 2363.088 by .8436.
2. 15552.12 by 7.358.
3. 161.8194 by 35.79.
4. 1795.830 by .5684.
5. 279.3056 by .05883.
6. 345.4858 by 42.63.
7. 5.518263 by 4.271.
8. 417.1076 by 753.8.
9. 69.93558 by 90.57.
10. 36816.06 by .8154.
11. 3082584 by 6381.
12. 272.3322 by 94.72.
13. 550226.1 by 8.495.
14. 5356615 by .5283.
15. 213.4242 by 524.6.
16. 41521.62 by 6.475.
17. 28.94448 by 816.7.
18. 379684.8 by .7384.
19. 18478.49 by .07496.
20. 2833110 by .4976.

21. There are 32.107 roses in a public garden, 712.417 in another, and 67.407 in a third; how many roses in the three gardens?

22. A carpenter bought 30 boards, each board contained 15 ft. and he gave $.07 a foot; what did the boards cost him?

23. A farmer bought 83.75 acres of land at $12.375, how much will he owe on his farm after paying $500?

24. If there are 375.75 lb. in a barrel of rice, how many lb. are there in 255 barrels?

25. If there are 63.75 acres in each of 80 fields, how many acres are there in all?

26. If one paving block weighs 16.48 lb., how much will 600 similar blocks weigh?

79. Special Rule.—Whenever the quotient is required to be in dollars and cents, annex ciphers to make the dividend contain two more decimal places than the divisor.

Exercises.

1. Divide $76847 by .79.
2. Divide 100,000 by .000001.
3. Divide .000001 by 1,000,000.
4. Divide 69,948 by .124.
5. $8764.87 by .1472.
6. 7.2 by .075.
7. 61.55 by .025.
8. $27.8 by .0016.
9. $.178 by 476.
10. $.796 by .25.

Problems.

1. If I pay $972.89 for 39 cows, how much does one cow cost?

2. How much coffee can be bought for $76.25 at 25 cents a pound?

3. How long will it take to go 768.91 miles, if a man can go 29.727 miles in one day?

4. How many bushels of wheat can be bought for $768, if one bushel costs 78 cents?

5. How many miles a day must a ship go, to pass round the world 30,000 miles in 179 days?

80. To reduce common fractions to decimals, and decimals to common fractions.

A fraction or a decimal is an unexecuted division, in which the numerator is the dividend and the denominator is the divisor.

Example 1.— Reduce $\frac{17}{24}$ to a decimal.

$$\begin{array}{r l} 24)\ 17.000\ (.708\frac{1}{3} & \\ 168 & \\ \hline 200 & \\ 192 & \\ \hline \frac{8}{24} = \frac{1}{3} & \end{array}$$

Rule.— Annex ciphers to the numerator and divide by the denominator.

Fractions should be reduced to some special decimal. The pupil should know what is required and how to obtain it. He should know how to find any given number of places.

Example 2.— Reduce .736 to a common fraction.

$\frac{736}{1000} = \frac{92}{125}$ **Rule.**— Remove the decimal point and write the decimal as a whole number for a numerator and 1 with as many ciphers as there are decimal places for a denominator, and then reduce the fraction to its lowest terms.

Exercises.

1. Reduce $\frac{69}{93}$ to thousandths.
2. Reduce .6748 to a common fraction.
3. Reduce $\frac{71}{147}$ to millionths.
4. Reduce .94864 to a common fraction.
5. Reduce $\$\frac{989}{1787}$ to cents.

Problems.

1. How many bushels in 9 sacks containing respectively .74 bu., $\frac{7}{8}$ bu., $\frac{9}{11}$ bu., .83 bu., $\frac{7}{12}$ bu., .88 bu., $\frac{10}{11}$ bu., .49 bu., and $\frac{11}{12}$ bu.

Reduce the common fractions to decimals.

2. How much more will a field containing 67$\frac{6}{7}$ A. at $49.63 an acre cost than another field containing 66.916 A.?

3. In one battle $\frac{1}{6}$ of the soldiers were disabled and .19 in another battle; if the army contained 42000 at first, how many remained effective after two battles?

4. At $\$\frac{3}{4}$ a yard, what will 75.84 yards of cloth cost?

5. A web of muslin containing $47\frac{7}{12}$ yards is to be made into garments for each of which 5.58 yards is required. After 8 garments are made, how much muslin is left?

6. A had 449 sheep, B had 186 more than A, and C had 89 more sheep than A and B together less 115; how many sheep had B and C each?

7. A farmer raised 1575 bushels of wheat and 1167 bushels of corn. He sold to D 879 bushels of wheat and 647 bushels of corn, and sold the remainder to H; how many bushels of each did H buy?

8. A man traveled 7689 miles; he traveled 2769 miles by railroad, 1642 miles in stage coaches, 736 miles on horseback and walked 79 miles. If he completed his journey by steamboat; how far did he travel by boat?

9. Three persons buy a hotel for $39,463. The first paid $11,469; the second $11,387, and the third paid the remainder; how much did the third man pay?

10. A man borrowed from his neighbor at different times $674, $892, $567 and $475 respectively.. If he paid him $2182, how much does he still owe?

11. If 116 tons of iron worth $65 a ton is required to build 1 mile of railroad, what will it cost for iron to build a railroad 289 miles long?

12. If a grocer buy 288 bbl. of flour for $1967, and sells it at $8.67 a barrel; how much does he gain?

Compound Denominate Numbers.

81. When a unit has a **name**, as 1 apple, it is called a **Concrete Unit.** One or more Concrete Units is a **Concrete** or **Denominate Number.** When a denominate number is expressed by **two** or **more** numbers of different denominations (names), it is a **Compound Number,** as: 4 lb. 7 oz.

82. Denominate numbers differ from abstract in having a **varying** instead of a **Decimal** scale: 1 ft.=12 in. The **Scale** is learned from the **Tables** below.

83. **Reduction** is the process of changing a number from one denomination to another without changing its value.

84. **Reduction Descending** is changing a number to an equal value of lower denomination, thus: 2 ft.=24 in.

85. **Reduction Ascending** is changing a number to an equal value of higher denomination, thus: 36 in.=3 ft.

AVOIRDUPOIS WEIGHT

86. Is used in weighing grain, groceries, coal, etc.

16 drams (dr.) = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
100 pounds = 1 hundred-weight (cwt.)
20 hundred-weight = 1 ton (T.)
1 ton = 20 cwt. = 2,000 lb. = 32,000 oz. = 512,000 dr.

32 lb.=1 bu. of oats.
56 lb.=1 bu. of corn.
60 lb.=1 bu. of wheat.
196 lb.=1 bbl. of flour.
200 lb.=1 bbl. of beef or pork.
80 lb.=1 bu. of bituminous coal.

Exercises.

1. How many ounces in 10 lb.? 15 lb.? 20 lb.?
2. How many lb. in 160 oz.? 320 oz.? 120 oz.?
3. How many bushels in 23,310 lb. of wheat?

4. How many barrels in 17,800 lb. of pork?
5. How many tons in 16,000 lb. of coal? 3,600 lb.?
6. Find the cost of 1,600 lb. of oats at 40 ct. per bu.
7. Find the cost of a load of coal weighing gross 4,400 lb. weight of wagon 1,200 lb. at 7½ ct. per bu.
8. How much will 17 lb. 10 oz. of beef cost at 7½ ct. a lb.?
9. At $67.89 a ton, how much will 1186 lb. of iron cost?

TROY WEIGHT

87. Is used in weighing precious metals and jewels.

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)
1 lb. = 12 oz. = 240 pwt. = 5760 gr.

Exercises.

1. How many ounces in 12 lb. Troy? 18 lb.? 30 lb.?
2. How many grains in 60 oz. Troy? 20 oz.? 70 oz.?
3. Find the cost of 3 pwt. of gold, at 5 cents per gr.
4. Find the cost of 3 lb. of silver, at 5 cents per oz.

APOTHECARIES' WEIGHT.

88. **Apothecaries' Weight** is used in weighing drugs and medicines when mixing and compounding them.

20 grains (gr.) = 1 scruple (℈)
3 scruples = 1 dram (ʒ)
8 drams = 1 ounce (℥)
12 ounces = 1 pound (℔)
1 ℔ = 12 ℥ = 96 ʒ = 288 ℈ = 5760 gr.

Exercises.

1. In 560 ℈ how many ounces? In 960 ℈? In 1200 ℈?
2. Change 5 ℥ to gr.; 7 ʒ to gr.; 3 ℔ to ℈.
3. How many more gr. in 1 ℔ than in 5 ℥?
4. Change 3½ ℥ to ℈; 4¼ ℈ to gr.

LIQUID MEASURE.

89. **Liquid Measure** is used in measuring liquids, such as liquor, molasses, vinegar, milk, etc.

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

1 hhd. = 2 bbl. = 63 gal. = 252 qt. = 504 pt. = 2016 gi.

1. How many pints in 40 gallons? in 5 bbl.?
2. How many bbl. in 4032 pints? in 8064 gi.?
3. I bought a barrel of cider for $3 and after it became vinegar I sold it at 12½ cents per qt.; find the gain.

DRY MEASURE.

90. **Dry Measure** is used in measuring grain, fruit, salt, nuts, etc.

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

1 bu. = 4 pk. = 32 qt. = 64 pt.

1. In 30 bu. how many pints? in 45 bu.?
2. In 2,560 pints how many pecks? how many bushels?
3. Change 20½ bushels to pints.
4. How much is made on a load of 51 bu. of oats at 40 ct. a bu., fed out to horses at 25 ct. for 4 quarts?
5. How much will 79 hogsheads of molasses cost at 39 cents a gallon?
6. At 18 cents a gallon, how many barrels of vinegar can be bought for $58.32?

TIME.

91. **Time is used in measuring periods of duration.**

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 year (yr.)
366 days = 1 leap year.
12 calendar months = 1 year.
30 days = 1 month (mo.)

1 yr.=12 mo.=365 da.=8,760 h.=525,600 min.=31-536,000 sec.

Thirty days have September, April, June and November. All the rest have thirty-one save February alone, which has twenty-eight and leap year twenty-nine.

92. The Division of the Year.

Days.	Months.	
31	Jan.	Winter.
28 (29)	Feb.	
31	Mar.	Spring.
30	Apr.	
31	May	
30	Jun.	Summer.
31	July	
31	Aug.	
30	Sept.	Autumn.
31	Oct.	
30	Nov.	
31	Dec.	Winter.

Leap Years.—Every year that is divisible by 4 is a leap year except centennial years which are leap years only when divisible by 400. The year 4,000 will not be a leap year.

1. Reduce 1,054,080 minutes to months; to years.
2. Reduce 6 years to hours; to seconds.
3. How much time will a man save in rising 30 minutes earlier every morning for 10 years?
4. How many more minutes in July than Feb., in 1884?

LONG MEASURE.

93. Long Measure is used in measuring distances.

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards = 1 rod (rd.)
320 rods = 1 mile (mi.)

$$69.161 \left\{ \begin{array}{l} \text{1 degree of latitude.} \\ \text{1 degree of longitude} \\ \quad \text{on the equator.} \end{array} \right.$$

360 degrees = 1 circumference of the earth.

1 mi. = 320 rd. = 1,760 yd. = 5,280 ft. = 63,360 in.

1. How many miles in 8,960 ft.? in 696,960 in.?
2. Change 7 miles to inches; 3 miles; 9 miles.
3. How many miles in 70 degrees of longitude on the equator?
4. How many yards in 10 miles? in 20 miles?

SQUARE MEASURE.

94. Square Measure is used in measuring surfaces; as of land, floors, walls, etc.

A square is a figure whose length and breadth are equal and its corners square.

Note.—Let pupils represent on the blackboard squares of various sizes as 1 in., 2 in., 3 in., 5 in., 12 in. on a side, and divide them into smaller squares of 1 in. on a side. Realize that the number of units of area equals the product of the linear units of the length and breadth.

Let the pupil develope the square measure table from the linear as far as to 1 square rod.

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

1 sq. mi. = 640 A. = 102,400 sq. rd. = 3,097,600 sq. yd. = 27,878,400 sq. ft. = 4,014,489,600 sq. in.

1. Find the number of square feet in a yard 6 rd. long and 5 rd. wide.

2. How many sq. yd. of paving in a yard 36 ft. long and 24 ft. wide? Find the cost at $1.50 per sq. yd.

3. How many sq. yd. in the floor of a room 18 ft. × 12 ft. Find the cost of carpeting it at $2.00 per sq. yd.

4. Find the cost of plastering the ceiling of the room in example 3 at 30 ct. a sq. yd.?

5. How many square yards in the floor of a room 40 feet long and 32 feet wide?

6. If a room is 30 feet long, 24 feet wide and 12 feet high, how many square yards in the walls and ceiling?

CUBIC MEASURE.

95. **Cubic Measure** is used in measuring anything that has length, breadth and thickness; as earth to be excavated, wood, walls, etc.

A cube is a body whose length, breadth and thicknes are equal and its corners all square.

Note.—Let pupils draw cubes with an edge 1, 3, 5, 12 inches and dividing into smaller cubes realize that the number of cubic units of the solids equals the product of the number of linear units in the three dimensions. Develope the cubic table as far as cubic yards.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
16 cubic feet = 1 cord foot (cd. ft.)
8 cord feet or
128 cubic feet = 1 cord of wood (cd.)
24¾ cubic feet = 1 perch of stone or masonry (pch.)

1 cu. yd. = 27 cu. ft. = 46,656 cu. in.

1. How many cubic feet in a room 20×18×11 ft.?

2. How many cubic inches in a block of marble 2 ft. long, 6 in. wide and 5 in. thick?

3. How much does it cost to excavate a cellar 30 ft. long, 20 ft. wide and 9 ft. high at 50 ct. a cubic yard?

4. How much will 7.125 lb. of meat cost at 12.5 ct. a lb.?

5. Reduce 15 cu. yd. to cu. ft.

6. I exchange 2 cwt. of flour at 2 ct. a lb. for muslin at 10 ct. a yard; how many yards do I get?

7. How much will 3 tons of wheat cost at $1.10 a bushel?

8. At 40 cents a foot, how much will 20 rods of fence cost?

9. How many cubic feet in 3 cords? in 2 cords? in 1½ cords?

10. Reduce 466,560 cu. in. to cu. yd.

11. If a man travels at the rate of 31.5 mi. a day, in how many days can he travel 204.75 mi.?

12. If I buy 10 A. of land at $150 an acre, and sell it at $2 a square rod, how much do I gain?

13. How many cords of wood in a pile 60 ft. long, 4 ft. wide and 6 ft. high?

14. How many minutes from 6 o'clock A. M. to 5 o'clock P. M.

15. How many lb. in 64 oz.? in 240 oz.?

16. A farmer sold 60 bags of wheat, each weighing 138 lb. at $1.16 a bushel; how much did he get for his wheat?

ENGLISH MONEY.

96. English Currency is used in Great Britian.

		U. S. Value.
4 farthings (far.)	= 1 penny (d.)	\$.02
12 pence	= 1 shilling (s.)	.24$\frac{1}{3}$
20 shillings	= 1 pound or Sov. (£.)	4.86$\frac{2}{3}$

£1 = 20 s. = 240 d. = 960 far.

1. Change £50 to pence,—to farthings.
2. How many shillings in 4,800 far.? How many £?
3. Change \$4,380 to £. \$5,840 to £.
4. How many dollars in £774? in £684?

97. U. S. MONEY.

10 mills (m.) = 1 cent (ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar (\$)
10 dollars = 1 eagle (E.)
1 E. = \$10 = 100 d. = 1,000 ct. = 10,000 m.

Note.—For coins see page 99.

CANADA MONEY.

98. Same as that of the United States.

1. Change 10,000 mills to dollars,—to Eagles.
2. Change \$100 to cents,—to mills.
3. How many nickels in \$5? in a double-eagle?
4. What is the value of a half eagle, 7 fifty-cent pieces and 90 dimes?
5. How many dimes are equal to 3 double-eagles?
6. How many mills in 6 cents and 5 mills?
7. How many mills in \$9.065?
8. Reduce 10,000 mills to cents; to dollars.
9. Reduce 749 mills to cents and mills.
10. Reduce 15,643 mills to dollars, cents and mills.

ADDITION AND MULTIPLICATION.

99. Compound Denominate Numbers are **added** and **multiplied** in the same manner as simple numbers, except that in reducing units from one denomination to another a **varying scale** is used instead of the **decimal scale.**

Add.

2 ℔	3 ℥	2 ʒ	2 ℈	11 gr.
3 "	5 "	7 "	2 "	7 gr.
5 "	7 "	6 "	2 "	8 gr.
11 ℔	5 ℥	1 ʒ	1 ℈	6 gr.

Multiply.

2 bu.	3 pk.	5 qt.
		7
20 bu.	1 pk.	3 qt.

Note.—Have pupils go over the work until they realize that the processes of addition and multiplication of compound denominate numbers are identical with those of simple numbers, observing that the units of each partial result are reduced to the next higher denomination by dividing by the numbers in the scale.

Exercises.

1. Add 3 bu. 3 pk. 7 qt.; 2 bu. 1 pk. 5 qt.; 2 bu. 2 pk. 6 qt.; 1 bu. 5 pk. 7 qt.

2. Multiply each of the numbers in the last example by 5.

3. Add the four products of example 2 and see whether their sum equals 5 times the answer to example 1.

4. Add 5℔ 2℥ 5ʒ 2℈; 4℔ 11℥ 6ʒ 2℈; 6℔ 8℥ 7ʒ 2℈; 2℔ 7℥ 6ʒ 1℈.

5. Repeat 2 da. 6 hr. 10 min. 18 sec. six times and add.

6. Multiply the number in example 5 by six and see whether this product is equal to the sum obtained.

7. Repeat 2 mi. 40 rd. 4 yd. 2 ft. 5 in. seven times and find the sum.

8. Multiply the number in example 7 by seven and see whether the product equals the sum obtained.

SUBTRACTION OF COMPOUND NUMBERS.

100. Compound Denominate Numbers are subtracted in the same manner as simple numbers, thus:

From	£56	15s.	8d.	20 gal.	2 qt.	1 pt.	2 gi.
Take	£10	18s.	10d.	5 gal.	3 qt.	1 pt.	3 gi.
	£45	16s.	10d.	14 gal.	2 qt.	1 pt.	3 gi.

Note.—Show pupils that in subtracting a greater from a less number of units of any denomination, the value of a unit of the next higher denomination is added to both the minuend and the subtrahend in the same manner as in simple numbers.

Exercises.

1. From 3 da. 8 hr. 30 min., subtract 2 da. 10 hr. 20 min. 15 sec.
2. Find the difference in time from June 12, 1864, to March 6, 1871.
3. How old to-day is the man who was born Feb. 22, 1832?
4. How long is it since the Declaration of Independence was signed?
5. Find the difference in the time from July 11, 1864, to March 4, 1867.

DIVISION OF COMPOUND NUMBERS.

101. Compound Denominate Numbers are divided in the same manner as simple numbers, thus:

5) 7 lb. 8 oz. 13 pwt. 18 gr.

1 lb. 6 oz. 10 pwt. 18 gr. Ans.

Note.—Have pupils realize that the remainder after each division is reduced to the next lower denomination in the same manner as in simple numbers.

Exercises.

1. Divide 13 bu. 3 pk. 6 qt. 1 pt. by 6, 7, and 9.
2. Divide 125 cd. 7 cd. ft. 14 cu. ft. by 9.
3. Divide 212 T. 3 hhd. 39 gal. 3 qt. by 23, and 30.
4. Divide 976 bu. 3 pk. 5 qt. by 15, and 24.

REDUCTION OF COMPOUND NUMBERS.

102. 1. Reduce 3 bu. 2 pk. 4 qt. to quarts.

3 bu.
4
14 pk.
8
116 qt. Ans.

Analysis.—3 bu. = 12 pk.; 12 pk. + 2 pk. = 14 pk.; 14 pk. = 112 qt.; 112 qt. + 4 qt. = 116 qt.

2. Reduce 5℔ 3℥ 2ʒ 2℈ to grains.
3. Reduce 5 gal. 3 qt. 1 pt. to gills.
4. Reduce 4 lb. 10 oz. 8 pwt. to grains.
5. Reduce 475 gi. to integers of higher denominations.

4) 475 gi.
2) 118 pt. + 3 gi.
4) 59 qt. + 0 pt.
14 gal. + 3 qt.
14 gal. 3 qt. 0 pt. 3 gi. Ans.

Analysis.—475 gi. divided by 4, the number of gills in 1 pt. = 118 pt. with a remainder of 3 gi.; 118 pt. divided by 2, the number of pt. in 1 qt., = 59 qt., with no remainder; 59 qt. divided by 4, the number of qt. in 1 gal. = 14 gal. with a remainder of 3 qt. The last quotient and the several remainders form the answer.

6. Reduce 16,780 gr. Troy, to integers of higher denominations.

7. Reduce 47,869 gr. Apothecaries' to pounds and the fraction of a pound.

Note.—Divide by the continued product of the numbers of the scale from gr. to lb, or by 5,760.

8. Reduce 16s. 10d. 3far., to the fraction of a £.

Note.—Reduce to farthings and divide by 960. Why?

9. Reduce 5℔ 3℥ 2ʒ 2℈ to ℔. and fractions of ℔.

10. Reduce the answers in examples 2, 3 and 4 to integers and fractions of the highest denomination in each example.

11. Change the fractional part of answers in example 10 to decimals.

12. Reduce 39 rd. 5 yd. 1 ft. 6 in. to the fraction of a mile. Change the answer to a decimal.

13. Reduce 15 sq. rd. 20 sq. yd. 6 sq. ft. to the decimal of an A.

14. Reduce 3 bbl. 8 gal. 3 qt. 1 pt. 2 gi. to the fraction of a hhd.; also to a decimal of a hhd.

103. 1. Change .828125 bu. and $\frac{53}{64}$ bu. to integers of lower denominations.

$$\begin{array}{rl}
.828125 \text{ bu.} & \qquad 4 \text{ pk.} \times \frac{53}{64} = 3\frac{5}{16} \text{ pk.} \\
4 & \\
\hline
3.312500 \text{ pk.} & \qquad 8 \text{ qt.} \times \frac{5}{16} = 2\frac{1}{2} \text{ qt.} \\
8 & \\
\hline
2.500000 \text{ qt.} & \qquad 2 \text{ pt.} \times \frac{1}{2} = 1 \text{ pt.} \\
2 & \\
\hline
1.000000 \text{ pt.} & \qquad 3 \text{ pk. } 2 \text{ qt. } 1 \text{ pt. Ans.}
\end{array}$$

Note.—Change the decimal or fraction of a bu. to pecks by multiplying by 4. Retaining the integer, reduce the decimal or fraction of a peck to quarts. In like manner, reduce the decimal or fraction of a quart to pints. The several integers will be the required answer.

2. Reduce .675 T. to pounds.

3. Reduce .788 A. to integers of lower denominations.

4. Find the cost of $\frac{7}{11}$ A. of land at $.44 per sq. rd.

5. Reduce $\frac{5}{8}$ of a week to integers of lower denominations.

6. Reduce 3 bbl. 17 gal. 3 qt. 1 pt. 1 gi., to the fraction of a hhd.

7. Reduce $\frac{3}{4}$ of 5 pwt. to the fraction of a pound.

8. Reduce $\frac{4}{5}$ of a cent to the fraction of an eagle.

9. Reduce 1 qt. 1 pt. 1 gi. to a fraction of a gallon.

10. Reduce $\frac{19}{1440}$ A. to lower denominations.

11. Reduce .625 pt. to the decimal of a bu.

12. A rectangular piece of land 24 rd. long and 20 rd. wide, contains how many acres?

13. How many spoons weighing 25 pwt. each, can be made from 4 lb. 7 oz. of silver? Troy weight.

14. What part of an acre is a town lot 75 ft. x 160 ft.?

15. Find the cost of a pile of wood 21 ft. long, $6\frac{1}{2}$ ft. high and 16 ft. wide at $6.75 per cord.

16. Find the cost of 8 oz. 12 pwt. of silver at $18 per lb.

17. Find the cost of 3 bbl. 6 gal. 3 qt. of brandy at $11.75 per gal.

18. A druggist bought 2 lb. 7 oz. 4 dr. opium at $60 per lb. and sold it at the rate of 3 cents per grain. What was his gain?

19. Change the denominate number in the last example to integers of the Apothecaries' table.

Note 1.—In changing from Avoirdupois table to either Troy or Apothecaries' table, reduce the amount to pounds and fractions of a pound, change the result to grains by multiplying by 7,000, and reduce the grains to integers of higher denominations in the required table.

20. Change 2 ℔. 4 ℥ 7 ʒ to equivalents in Troy weight.

21. Change 3 lb. 3 oz. 8 pwt. 10 gr. to equivalent weight of the Avoirdupois table.

Note 2.—In changing from Apothecaries' or Troy to Avoirdupois, reduce to grains by the descending scale, and to pounds and fractions of pounds avoirdupois by dividing the grains by 7,000; then reduce the fraction to integers of lower denomination.

22. Change 2℔ 3℥ 7ʒ 1℈ to Avoirdupois weight.

23. Change 5 lb. 3 oz. 8 pwt. 12 gr. to Apothecaries' weight.

24. Change 1℔ 2℥ 4ʒ 2℈ 8 gr. to Troy weight.

Suggestion.—As pounds and ounces in Troy and Apothecaries' tables are the same, it is necessary to reduce only the denominations below ounces to grains and change to higher integers, according to the table of the required weight.

25. How many rings of 2 pwt. 8 gr. can be made from 7 oz. gold?

26. A watch gains 1 minute every four days, how much will it gain in the months of July and August?

27. A bicycle wheel is 12 ft. 6 in. in circumference, how many times will it revolve in going 3 miles?

28. Six men can do a piece of work in 12 days, how long will it take 9 men to do the same work?

29. How many squares 3 in. on a side may be cut from a sheet of card board 5 ft. by 4 ft.?

30. How many silver spoons can be made from 1 lb. Avoir. of silver, if there are 16 pwt. 8 gr. in each spoon?

31. How many village lots, each 66 ft. by 165 ft. can be laid out from a field of 4 A.?

32. A man was born Feb. 22, 1732, and died Dec. 14, 1799, how old was he?

33. A field containing 80 acres is 40 rd. wd., how long is it?

34. How long is a pile of wood containing $3\frac{1}{2}$ cords, if the width is 8 ft. and the height 6 ft.?

35. How many miles in 178,964 ft.? In 168,965 in.?

Relation of Numbers.

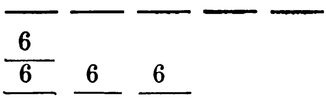
104. There are three operations under the relation of numbers:

I. Finding the fractional part of a number.

II. Finding the entire number when a fractional part is given.

III. Finding what part one number is of another.

I. 1. Find $\frac{3}{5}$ of 30.



$\frac{5}{5}=30$

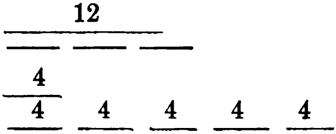
$\frac{1}{5}=\frac{1}{5}$ of $30=6$

$\frac{3}{5}$ of $30=6\times3=18$.

Analysis.—$\frac{1}{5}$ of 30 is 6; $\frac{3}{5}$ of 30 is $6\times3=18$. Ans.

2. Find $\frac{4}{7}$ of 28; of 35; of 42; of 50.

II. 1. 12 is $\frac{3}{5}$ of how many?



$\frac{3}{5}=12$

$\frac{1}{5}=\frac{1}{3}$ of 12 or 4

$\frac{5}{5}=4\times5=20$.

Analysis.—Since 12 is $\frac{3}{5}$ of a number, $\frac{1}{3}$ of 12 or 4, is $\frac{1}{5}$ of the number, and the entire number is 4×5, or 20. Ans.

2. 16, 24, 36 and 48 are respectively $\frac{4}{7}$ of what numbers?

3. 15, 21, 25 and 32 are respectively $\frac{3}{8}$ of what numbers?

4. $\frac{3}{4}$ of 24 is $\frac{3}{7}$ of what number?

5. $\frac{3}{5}$ of $3\frac{3}{4}$ is $\frac{4}{7}$ of how many?

6. $\frac{2}{3}$ of $4\frac{1}{2}$ is $\frac{3}{5}$ of how many times five?

7. $3\frac{1}{2}$ times $\frac{2}{3}$ of $5\frac{1}{4}$ is $2\frac{1}{3}$ times what number?

III. 1. 3 is what part of 12? $\frac{3}{12}$, or $\frac{1}{4}$, Ans.

Analysis.—3 is such a part of 12 as 12 is contained times in 3, or $\frac{1}{4}$.

Rule.—To find what part one number is of another, use the number compared as the numerator and the number with which it is compared for the denominator of a fraction; this fraction will express the fractional part that the upper number is of the lower.

2. 7 is what part of 14? of 28? of 35?

Problems.

1. $\frac{2}{3}$ of 15 is what part of 30?
2. $\frac{2}{3}$ of 12 is $\frac{4}{7}$ of what number?
3. $\frac{3}{5}$ of 10 is $\frac{5}{11}$ of how many times 6?
4. $\frac{3}{5}$ of 10 is $\frac{2}{7}$ of how many times $\frac{2}{3}$ of 15?
5. What part of $\frac{4}{7}$ of $5\frac{3}{5}$ is $\frac{4}{7}$ of $8\frac{3}{4}$?
6. 12 sq. rd. is what part of an A.?
7. Find how many sq. rd. in $\frac{4}{5}$ of an acre.
8. A pile of wood 4 ft. long, $1\frac{1}{2}$ ft. high and 2 ft. wide, is what part of a cord?
9. A gallon of water is 231 cu. in.; what part of a cubic foot is it?
10. How many cu. in. in $\frac{5}{12}$ of a cu. ft.?
11. Find the value of 10 A., 600 sq. rd. of land @ $40 per A.; @ $80 per A.
12. What part of a pound is 7 oz.?
13. Find the cost of 12 lb. 6 oz. of silver @ $12 per lb.
14. What part of a bu. are 3 pk.?
15. Find the cost of 65 bu. 3 pk. of wheat @ 80c. per bu.
16. How many cd. ft. in $\frac{7}{8}$ cd.?
17. If there are 63 gal. in a hhd., how many gal. are there in 7 hhd.

19. 40c. is what fractional part of $1? 60c.? 80c.? 37$\frac{1}{2}$c.?

20. Find $\frac{3}{4}$ of $1; $\frac{7}{8}$, $\frac{1}{6}$, $\frac{1}{16}$, $\frac{1}{25}$.

21. Write twenty fractional parts of a dollar.

Write the value of the following articles:

22. 1650 lb. of bacon, @ 16$\frac{2}{3}$c. per lb.

23. 1850 lb. of cheese, @ 12$\frac{1}{2}$c. per lb.

24. 1760 lb. of sugar, @ 6$\frac{1}{4}$c. per lb.

25. 2400 lb. of coffee, @ 37$\frac{1}{2}$c. per lb.

26. 1600 lb. of tea, @ 62$\frac{1}{2}$c.

105. 1. What is the value of 3600 fence posts, @ $17 per hundred? @ $140 per 1000?

$17	$140
36.00	3.600
$612.00. Ans.	$504.00. Ans.

Rule.—Multiply the price per 100, or 1000 by the number of articles reducd to hundreds or thousands.

2. What is the freight on 217600 lb. of iron, @ 60c. per 100 lb.? @ $5 per 1000 lb.?

3. Find the cost of 6780 shingles @ $3.60. M. (1000.) @ $ .40 per C. (100.)

4. Find the value of 6780 lb. of hay, @ $16 per ton.

Note.—Divide the price per ton by 2 to get the price per 1000 lb.; then work according to the Rule.

5. What will be the freight on 6789 lb. of iron, @ $2.60 per ton?

6. What will 78,690 lb. of iron cost, @ $27.50 per T.?

7. Find the cost of 3 loads of straw, each weighing 2,275 lb., at $6 per ton.

8. A load of coal and the wagon weigh 5,680 lb.; if the wagon weighs 1,480 lb., what is the value of the coal at $5 per ton?

106. 1. Copy and complete the following bill:

MR. JOHN S. JONES. St. Louis, Feb. 12, 1886.

To SMITH, PETERSON, BROWN & CO., DR.

1885.				
July.	15	To 16 yd. blk. Grenadine, @ 1.40	$	
"	"	To 16 " " Silk " 1.00		
Sept.	23	To 15 " garnet Silk, " 2.50		
Dec.	21	To 18 " blk. Cashmere, " 1.50		
"	"	To 4 " " Wtrd Silk, " 3.75		
"	28	To 6 PR. fancy silk Hose, " 3.50		
"	"	To 3 " blk. kid Gloves, " 2.50		
			$	

Received payment,
SMITH, PETERSON, BROWN & CO.

2. Make out the following bill:

MR. J. Y. LINDSEY, New York, Dec. 18, 1888.

To JOUNNEY, BURNHAM & CO., Dr.

1888, Oct. 10.—To 10 yd. Table Linen, at $1.50; 2 doz. Dinner Napkins, at $2.50. Oct. 12.—16 yd. Calico, at 10c. Oct. 19.—6 pr. Balbriggan Hose, at 50c. Nov. 14.—5 yd. Opera Flannel, at $1.00; 12½ yd. Shaker Flannel, at 25c. Dec. 1.—15 yd. Unbleached Muslin, at 10c.; 2 yd. English Silesia, at 18c. Dec. 9.—4 pr. Children's Hose, at 75c. Dec. 17.—12 yd. Tycoon Reps., at 25c.

107. 1. Find the cost of plastering the walls and ceiling of a room 18 feet long, 15 feet wide, 10 feet high, at 33 cents per square yard.

18ft.+18ft.+15ft.+15ft.=66 ft. the length of one wall, which is equal to the four walls of the room.

$\frac{66 \times 10}{9} = 73\frac{1}{3}$ square yards in the walls.
$\frac{18 \times 15}{9} = 30$ square yards in the ceiling.

$\$.33 \times 103\frac{1}{3} = \$34.10$. Ans.

Note.—Plastering and painting are usually estimated at so much per square yard, allowance for openings being made as per contract.

Show the pupils by folding paper that the four walls of a room can be considered as one straight wall whose length is the sum of the four sides of the room.

2. Find cost of kalsomining the same room at 3 cents per square yard.

3. Find cost of painting a board fence 8 feet high around a field 40 yards long by 30 yards wide, at 5 cents per sq. yd.

108. 1. Find the cost of papering a room 20ft.x18ft., 8½ ft. high, at 25 cents per roll on the wall and 30 cents per roll on the ceiling, the border costing 40 cents per roll of 8 yards.

$\$\frac{1}{4} \times \frac{76 \times 17}{2} \times \frac{1}{9} \times 2 \times \frac{1}{8} = \$4.49$, cost of papering wall.
$\$\frac{3}{10} \times \frac{20 \times 18}{9} \times 2 \times \frac{1}{8} = 3.00$, cost of ceiling.
$\$\frac{4}{10} \times \frac{76}{3 \times 8} = 1.26$, cost of border.

$\$8.75$. Ans.

Note.—A roll of paper is usually 8 yd. long and half a yd. wide. The waste from matching the figure will usually be made up from what is saved from the openings.

2. Find the cost of papering a house of six rooms, each 12x15 feet, 9 feet high, with paper 50 cents per roll on the walls and 60 cents on the ceiling, border 75 cents per roll. Make allowance for 12 doors and 18 windows, each 7 feet by 3½ feet.

3. Find the cost of graining and varnishing the doors in example 2, at 25 cents per square yard.

4. Find the cost of 68,580 lb. of coal at 11 cents per bushel of 80 lb.

5. Find the cost of 6 planks, 16 feet long, 10 inches wide, 2½ inches thick, at $18 per 1,000 feet,

6. Find the cost of 380,000 lb. R. R. iron, at $24 per ton.

7. How much will it cost to build 100 yards of fence at 5 cents per foot?

8. Find the cost of papering a room 20 feet long, 14 feet wide, 9 feet high, with paper at 40 cents per roll on the wall and 45 cents on ceiling, allowing 1½ rolls for doors and windows; border costing 60 cents per roll of 8 yards.

109. 1. Find the cost of building the foundation wall of a house 40 feet long, 23 feet wide, at $3.75 per perch, the wall being 11 feet high and 18 inches thick.

40 ft. + 40 ft. + 23 ft. + 23 ft. = 126 ft. distance around the walls, or girth of building.

$126 \times 11 \times \frac{3}{2}$ = cubic feet in wall.

$126 \times 11 \times \frac{3}{2} \times \frac{4}{99}$ = number of perch of masonry.

$\$\frac{15}{4} \times 126 \times 11 \times \frac{3}{2} \times \frac{4}{99} = \$315$ cost of masonry.

2. Find the cost of building the foundation wall, 14 ft. high and 18 in. thick, of a house 60 ft. by 50 ft., at $4.25 per perch.

110. 1. Find the cost of building the walls of a house 30 ft. by 20 ft., 24 ft. high, 13 in. thick, of brick, at $8 per 1,000 brick.

100×24 = superficial feet of walls.

$21 \times 100 \times 24$ = number of bricks.

$$\frac{\$8 \times 21 \times 100 \times 24}{1{,}000} = \$403.20 \text{ cost, or Ans.}$$

NOTE.—For every superficial foot of walls, builders allow 7 bricks for a four inch wall, 14 bricks for a nine inch wall, 21 bricks for a thirteen inch wall, etc., adding 7 bricks for each 4½ inch additional thickness.

2. Find the cost of building the walls of a house 120 feet long, 45 feet wide, 30 feet high, walls 26 in. thick; allow 40 square feet additional for the gables and deduct ½ for 6 doors, 6 ft. long and 3 ft. wide, and 30 windows, 8 ft. long and 3 ft. wide; bricks worth $9 per 1,000.

3. Find the cost of painting the house in example 2, at 50 cents per square of 100 feet; no allowance for openings.

4. Find the cost of shingling the house in example 2, if the length of rafters is 30 feet and the ends of roof project 18 in.; shingles $3.00 per 1,000, each shingle being 4 in. wide and laid 6 inches to the weather.

111. 1. Find the cost of carpeting a room 15½ feet wide, 20 feet long with carpet, 27 in. wide, at $1.25 per yard, laid the long way, allowing 6 inches for match of figure and for turning under any extra width.

Suggestion.—Length of strip of carpet 20½ feet, number of strips 7, with 3 inches turned under at the side.

$\frac{41}{2} \times 7 \times \frac{1}{3} =$ number of running yards.

$\$\frac{5}{4} \times \frac{41}{2} \times 7 \times \frac{1}{3} = \$59.79$ Ans.

2. Find the cost of carpeting a room 13½ feet wide, 16 feet long, with carpeting 27 in. wide, worth 80 cents per yard; allow 4 inches for match of figure, and lay the carpet the long way of the room.

3. Find the cost of carpeting a room 30ft. by 18ft., with carpeting 1 yard wide, at $1.59 per yard, allowing no waste.

112. 1. Find the cost of flooring ⅞ inch thick, for a room 30 ft. by 20 ft., at $35 per 1,000 board ft.

Note.—A board foot is a surface 1 ft. square and an inch (or less) thick. When the lumber is more than 1 inch thick, as many times the the superficial feet should be taken as the lumber is inches in thickness.

$$\frac{\$35 \times 30 \times 20}{1,000} = \$21. \text{ Ans.}$$

2. What is the cost of flooring, 1½ inch thick, for 3 rooms, each 30 ft. by 18 ft., at $32 per 1,000 ft. board measure?

3. Find the cost of flooring 2½ in. thick, for a stable 18 ft. by 12 ft., at $18 per 1000 board feet.

113. 1. How many bushels and how many gallons Liquid Measure will a tank 8 feet square and 7 feet deep contain?

$$\frac{8 \times 8 \times 7 \times 1728}{2150.42} = ? \qquad \frac{8 \times 8 \times 7 \times 1728}{231} = ?$$

Rule.—Divide the cubical contents in inches by 2150.42 to reduce to bushels, or by 231 to reduce to wine gallons.

2. How many bushels of wheat in two bins each 20 ft. long and 3 ft. wide, the wheat being 3 ft. 6 in. deep in one and a foot deeper in the other?

3. A dealer buys chestnuts at $3 a bushel and sells them at 5 cents per half-pint wine measure; how much does he gain in selling 5 bushels?

4. How many bu. are equal in bulk to a cord of wood?

5. How many cords in a pile of wood 20 ft. long, 4 ft. wide, and 8 ft. high?

6. A house contains 6 rooms, each 16 ft. by 12 ft. and 10 ft. high. Find the cost of plastering the same at 33 cents per square yard.

7. What will it cost to carpet the rooms in example 6, with carpeting 27 in. wide at 80 cents per yard; no allowance for waste?

PERCENTAGE.

Definitions.

114. In an orchard of 600 trees, 10 out of every 100 are peach trees; how many peach trees are there? 12 out of every 100 are apple trees; how many apple trees are there? 60 out of every 100 are pear trees; how many pear trees are there? The rest of the trees are cherry; how many out of each 100 are cherry trees? How many cherry trees are there in the orchard?

115. If there are 1200 trees in an orchard and 8 out of 100 are peach, 20 out of 100, apple, 40 out of 100, pear, and the rest cherry, how many of each kind of trees are there?

116. 8 out of 100 is equivalent to $\frac{8}{100}$, .08 or 8 per cent.

117. **Per cent.** means by the 100, and may be written in words or represented by the character %.

Note.—Let the pupil write example 1 above, using the words per cent. instead of 10 out of 100, etc.; and example 2, using the symbol % instead of 8 out of 100, etc.

118. Express 5%, 10%, $12\frac{1}{2}$%, 30%, $6\frac{1}{4}$% and $37\frac{1}{2}$% as a decimal, also as a common fraction.

119. Express $\frac{1}{4}$%, $3\frac{1}{3}$%, $\frac{1}{5}$%, $\frac{1}{25}$% and $\frac{1}{30}$% as a common fraction.

Note.—$1\% = \frac{1}{100}$, $3\frac{1}{3}\% = \frac{1}{100} \times \frac{10}{3} = \frac{1}{30}$.

Problems.

120. 1. Find 5% of 400; of 360; of 285; of 1670.

Suggestion.—Find $\frac{5}{100}$, or $\frac{1}{20}$ of $400 = 400 \times .05 = 20$. Or, $400 \times .05 = 20.00$, Ans.

2. Find 20%, 37½%, 26¼%, $\frac{1}{25}$% of 6800.

3. A man has a farm of 60 acres and sows 20% of it to wheat and 10% of the remainder to oats; how many acres of oats does he sow?

4. In an army there are 100,000 men; 12½% of them were killed in battle; 20% of the remainder were wounded; how many remained for service?

5. A man has 600 acres of land and sells 20% of it; how many acres does he sell? How many does he have left? Under which of the relations of numbers do these operations come? See page 123, §104.

6. Express fractionally and decimally 6%;—11%;—14%;—21%;—35%;—48%;—75%.

7. What is 5% of 400?—of 1,000?—of 2,500?—of 3,000?—of 10,000?

8. What is 12% of 600?—of 900?—of 3,600?—of 4,200?—of 5,400?

9. What is 4% of 125 gal.?—of 50 hhd.?—of 500 bu.?—of 7,000 lb.?—of 340 mi.?

10. What is 8% of $400?—of $850?—of $960?—of $1,845?—of $7,500?

11. What is 20% of 40 A.?—of 320 A.?—of 640 A.?—of 80 A.?—of 2,000 A.?

12. What is 50% of 540 lb.?—of 78 T.?—of 420 oz.?—of 780 lb.?—of 520 T.?

13. What is 42% of $960?—of 1,865 ft.?—of 465 gal.?—of 85 A.?—of 780 bu.?

121. 1. 20% of an orchard is in apple trees; if there are 80 apple trees, how many trees in the entire orchard?

20%=$\frac{1}{5}$ or .20. 80 is $\frac{1}{5}$ or .20 of how many? 80÷$\frac{1}{5}$=400. 80÷.20=400. Ans.

Under which of the relations of numbers does this operation come? See page 123, § 104.

2. 60 is 3% of how many? 180 is $\frac{1}{4}$% of how many?
3. 120 is 10% of how many? 36 is $37\frac{1}{2}$% of how many?
4. 140 is 5% of how many? 800 is $16\frac{2}{3}$% of how many?
5. 400 is $\frac{1}{2}$% of how many? 1,000 is $6\frac{1}{4}$% of how many?
6. What number increased by 25% of itself equals 600?

600 is 125% or $\frac{5}{4}$ of the number. $600 \div \frac{5}{4} = 480$. Ans.

7. What number increased by 30% of itself equals 520?
8. What number decreased by 20% of itself equals 1,600?
9. 48 is 20% more than what number?
10. 630 is 10% less than what number?
11. 20% of a cask of wine has been drawn, and there are 36 gallons left; how many gallons were drawn?

122. 1. 12 gallons are what part of 48 gallons? What per cent.?

$\frac{12}{48} = \frac{1}{4} = 25\%$; or 12 gal. ÷ 48 gal. = .25 or 25%. Ans.

Under which of the relations of numbers does this operation come? See page 123, § 104.

2. A man having 60 acres of land, sold 20 acres; what part and what per cent. of his farm did he have left?

3. 20 days are what per cent. of one year?

$365)20.00(.05\frac{35}{73}$ or $5\frac{35}{73}\%$ Ans.
1825

175

$\frac{175}{365} = \frac{35}{73}$

Note.—In finding the fractional part or per cent. of a number, carry the quotient to two decimal places and express the remainder by a fraction.

4. 1 pint is what per cent. of $1\frac{1}{2}$ gallons?

5. 3 quarts is what per cent. of $\frac{1}{2}$ bushel?

What per cent. of—

6. $300 are $27? 7. $25 are $6.25? 8. 4,800 are 168?

9. $120 are $4.50? 10. 1,285 are 25? 11. $400 are $16?

12. A man earns $900 a year; his expenses are $37.50 a month. What per cent. of his salary can he save?

13. What per cent. of $105 are $12\frac{1}{2}\%$ of $480?

14. A man's property is worth $8,000; he owes $24,000; what per cent. of his debts can he pay?

15. A field containing 40 sq. rd. is what per cent. of a field 40 rd. square?

16. A boy bought a sled for 80 ct. and sold it for $1.20; what per cent. did he gain?

17. A building valued at $5,000 was insured at $2\frac{5}{8}\%$; find the premium or cost of insurance.

18. How shall I mark goods costing 75 ct. to gain 10%?

19. How shall I mark goods costing $4.00 to gain $37\frac{1}{2}\%$?

20. An agent sold a house for $850, at a commission of $2\frac{1}{2}\%$; what was his commission?

21. What is the selling price and gain on goods that cost $2,000, that were sold at a profit of 30% of the cost?

22. Sold $\frac{5}{8}$ of a mine for $6,400: what was the value of the remainder at the same rate?

23. A man spent 25% of his money and then had $48; how much had he at first?

24. A ton of hay cost $24; what part of a ton would $16 buy?

25. Paid $9.60 for potatoes and sold them for $10; what per cent. of the cost did I gain?

26. A merchant drops 25% from the price he asks for goods, and sells them for $33, gaining 50%; what would he have gained if he had sold them at his asking price?

27. An agent sold a house for $680, and sent the owner the proceeds after taking out his commission at 2%; what did the owner receive?

28. An agent sells a farm for $6,000, receiving 1½% of the sales as his commission; how much is his commission and how much does he remit to the owner?

29. A man sells his house through an agent for $12,000, paying 1¼% commission; how much does he realize for his house?

30. A farmer sends his crop of wheat, consisting of 10,000 bushels, to a commission merchant to sell on a commission of ½%; the merchant's commission is $45; how much does the farmer receive for his wheat? How much per bushel?

31. An agent sells a piano for $400, after making a discount from the retail price of $50, and makes $50 profit; what was his per cent. of commission on the retail price?

32. A sewing-machine agent is allowed 40% on his $80 machines, but makes a discount of 20% to his customer; what is his profit on the machine?

33. $60 cost, $40 gain; find the selling price and per cent. of gain.

34. $80 selling price, $70 cost; find the gain and per cent.

35. $4,000 amount of sales, $60 commission; find the rate of commission and amount realized by the owner.

36. $8100 amount realized by owner, rate of commission 10%; find amount of sales and commission.

37. If a man's annual income is $6,000 and he pays an income tax on all over $4,000, at the following rates; 2% on the first thousand and 3% on the next, what will be his income tax?

38. A man paid $440 for a horse and carriage paying 20%

more for the horse than for the carriage, how much did he pay for each?

39. A commission merchant received 2% commission on a certain sale of wheat and 3% on a sale of corn amounting to twice as much; his entire commission was $800, find the amount of each sale.

40. At the end of a year 28% of a merchant's sales has been profit; what per cent. has he gained on the cost of his goods?

41. A man's salary was increased 25% at the end of his third year's engagement; the following year he received $6,000; what was his third year's salary? What per cent. less was his third years' salary than his fourth year's?

42. John is 20% older than James; what per cent. is James' age less than John's?

43. A father divides $42,000 among his three sons giving the first $\frac{1}{3}$ more than the second and the third $\frac{1}{2}$ as much as the first and second; how much has each?

44. A wholesale merchant makes a discount of 20% from the list price to the retail trade and 5% off for cash; what does the retailer pay in cash for 600 wagons at $30 each?

45. What will be the nét amount of a bill of glass, listed at $600, but with the following discounts allowed, 30%, 20%, 10% and 5% off for cash?

46. An insurance agent makes out each month ten policies for $5,000 each, at a premium of 2% of the amount of the policies; if the agent gets 60% of the first premiums, what is his monthly income?

47. What will it cost to insure a house for 5 years for $6,000 at $\frac{3}{5}$% for the term?

48. Find the cost of $12,000 insurance for 6 years at $\frac{3}{4}$% for the term.

49. If it cost \$18 to insure a house for \$3,000 for 5 years, what is the rate?

50. Insurance of \$12,000 cost \$48 per year; what per cent. is it? What should be the rate for 5 years?

51. If a man's property is worth \$40,000, and he pays \$800 taxes, what per cent. does he pay?

52. A man gave his son \$6.25 which was $\frac{5}{7}$ of as much as he had after the gift; how much had he at first?

53. 7 is what per cent. more than 5?

54. 5 is what per cent. less than 7?

55. 7 is what per cent. of 5?

56. The sum of 7 and 5 is what per cent. of each? Each is what per cent. of their sum?

57. William has 20% less money than Joe; the difference between their moneys is 10 cents; how much has each?

58. Corn is bought for 64 cents a bushel and is sold for 52 cents a bushel; what is the rate of loss?

59. A farmer sold a cow which cost \$39 for \$52; what was his rate of gain?

60. How much will a man lose who sells two coats for \$50 each, if he makes 25% on one and loses 25% on the other?

61. If wheat costs 75 cents a bushel and is sold at 80 cents a bushel, what is the per cent. of gain?

62. What rate per cent. of gain will a miller receive, if he sells flour at \$7 per barrel, and pays \$4 for the wheat out of which he made it?

63. A merchant bought carpets at 70 cents a yard which he sold at \$1.10 a yard; what is his rate of gain?

64. How much per cent. will be gained by buying flour at \$5.50 per barrel and selling it at \$6.75 per barrel?

65. A farm was bought at \$40 per acre and sold at \$48 per acre; what was the per cent. of gain?

Stocks and Bonds.

123. When stocks and bonds are worth more than their face or par value, they are at a **premium** or **above par**; when they are worth less than their face or par value, they are at a **discount** or **below par.**

124. The broker's fee for buying or selling stocks or bonds is usually $\frac{1}{8}\%$ for each transaction, and is added to the market value in buying and subtracted from the market value in selling.

125. The face value of a bond is usually $100, $500 or $1,000. The face value of a share of stock is $100, unless a different amount is named.

126. 1. Find the cost of 60 shares of Mo. P. stock at $110, or 10% premium, brokerage $\frac{1}{8}$.

$100×60=$6,000, par val. $6,000
$100\%+10\%+\frac{1}{8}\%=$ 1.10125

$6,607.50. Ans.

Problems.

2. Find the selling price of 80 shares canal stock at 10% discount, brokerage $\frac{1}{8}$. ($100\%-10\%-\frac{1}{8}\%=.89\frac{7}{8}$.)

3. What will be the gain on 60 shares of stock bought at 120 and sold at $124\frac{1}{2}$, brokerage $\frac{1}{8}$ each way?

4. A man exchanged 80 shares St. P. at 120 for 300 Erie at $16\frac{1}{2}$, brokerage $\frac{1}{8}$ each way; did he pay or receive the difference, and how much?

5. How much do I lose on 500 shares of C. B. & Q. stock that cost 86, when I sell it for 79, brokerage $\frac{1}{8}$ each way?

6. If I bought 200 shares of Lake Shore stock at 126, giving in part payment 140 shares of St. Paul at 120, brokerage $\frac{1}{8}$ each way, how much cash was needed to pay the balance? Do I receive or pay the cash?

7. A man sold 300 shares of N. Y. & N. E. at 23, which cost 26; how much does he gain or lose if the brokerage was $\frac{1}{8}$% each way?

8. How many Mo. 6's can be bought for $33,075 if they are at 10% premium, brokerage $\frac{1}{4}$? What will be the income on the bonds at 6%?

9. How much income will be realized from an investment of $6,000 in bank stock at 150, if the stock pays a dividend of 3% four times each year?

127. Bonds are usually named according to the rate of annual interest they bear; thus,—3's, 4's, 6's are names for bonds yielding 3, 4 and 6 per cent. interest respectively per annum.

What will be the income from—

1. $6,000 of 6's? 3. $80,000 of 8's? 5. $10,000 of 4$\frac{1}{2}$'s?
2. $12,000 of 4's? 4. $7,000 of 3$\frac{65}{100}$'s? 6. $8,000 of 8's?

128. 1. How many 4's, 6's, 8,s or 10's, respectively, must be bought to secure an income of $480?

2. How much will each of these amounts of stock cost if bought at 80, brokerage $\frac{1}{8}$? At 120, brokerage $\frac{1}{8}$?

129. 1. When 4% stocks or bonds are bought at 60, 80, 120, respectively, what rates will be realized on the investment?

Suggestion.—$.04, income on $1, ÷ $.60, cost of $1 of stock = .06$\frac{2}{3}$ or 6$\frac{2}{3}$%. $.04 ÷ $.60 = .06$\frac{2}{3}$ or 6$\frac{2}{3}$%.

2. When 6's are bought at 60, 80, 90 and 120, respectively, what rates will be realized?

130. 1. At what prices must 6's be bought to realize 5%, 8%, 10% and 12%, respectively, on the investment?

Suggestion.—\$.06, income from \$1 of stock, is 5% of cost. \$.06÷.05=\$1.20, or market value of \$1.00 of stock.

2. If the stock of an insurance company sell at 6 per cent. below par, what will \$1,200 of the stock cost?

3. What is the market value of 35 shares of Panama Railroad stock at 18% below par?

4. How many shares of railroad stock, at 5% advance can be purchased for \$12,600?

5. I invested \$12,375 in Ocean Telegraph stock, at 10% discount; how much did I purchase?

6. If I invest \$1,632, in 6% bonds at 102, what income will I receive?

7. What will be my yearly income if I invest \$4,214 in 7's at 98?

8. If N. Y. 6's are 20% below par, what sum must we invest to obtain an income of \$840?

9. What sum must be invested in 5's at 98¼%, brokerage ¼% for buying, to secure an income of \$930?

10. When N. Y. 5's are quoted at 108¼, what sum must I invest to secure an annual income of \$540, brokerage ¼?

11. What must be the price of stock in order that \$12,500 stock may be bought for \$8,750?

12. What income will be derived from \$40,000 of 5% bonds?

13. Find the income on \$36,400 invested in 4's at 91.

14. Find the income on \$6,150 invested in 6% stock at 102½.

15. How much money must be invested in a 4% stock at 87½, to produce an income of \$300?

16. A man buys some 4% railroad stock at 75, and receives \$800 income. How much money does he invest?

Interest.

131. **Interest** is a sum paid for the use of money.

132. **Principal** is the money for which interest is paid.

133. A man borrowed \$600 agreeing to pay 6% of the principal each year for the use of the money; how much per year did he pay? How much would he pay in 3 years? A man borrowed \$350 agreeing to pay 7% of the principal each year for its use; how much would he pay in 5 years?

134. **The rate** is the per cent. of the principal which is paid each year. The **time** is the number of years, or part of a year for which interest is paid.

135. The **amount** is the sum of the principal and interest.

136. 1. Find the interest of \$60 at 10% per annum for 3 yr. 7 mo.

$$\begin{array}{r l}
 & \$60. \\
 & \underline{.10} \\
12) & \$6.00 \times 3 = \$18.00 \text{ interest for 3 yr.} \\
 & .50 \times 7 = \$\ \ \underline{3.50} \text{ interest for 7 mo.} \\
 & \qquad\qquad \$21.50. \text{ Ans.}
\end{array}$$

Give Analysis.

2. Find the interest of \$75 at 7% for 3 yr. 5 mo.
3. Find the interest of \$80 at 9% for 3 yr. 3 mo.
4. Find the interest of \$90 at 19% for 2 yr. 11 mo.
5. Find the interest of \$125 at 12% for 3 yr. 2 mo.
6. Find the interest of \$1 for 7 yr. 8 mo. at 5%? — at 6%? — at 7%? — at 8%? — at 10%?
7. What is the interest of \$749 for 4 yr. 3 mo. at 5%?
8. What is the interest of \$1,680 for 3 yr. 7 mo. at 7%?
9. Find the amount of \$600 for 2 yr. 9 mo. at 5%?

137. 1. Find the interest of $360 at 5% for 3 yr. 8 mo. 12 days.

$$
\begin{array}{rl}
 & \$360 \\
 & \underline{.05} \\
12) & \underline{18.00} \times 3 = \$54.00 \text{ interest for 3 years.} \\
30) & \underline{1.50} \times 8 = 12.00 \text{ interest for 8 mo.} \\
 & .05 \times 12 = \underline{.60} \text{ interest for 12 da.} \\
 & \ \$66.60 \text{ interest for 3 yr. 8 mo. 12 da.}
\end{array}
$$

Find the interest and amount of

2. $760 for 3 yr. 6 mo. 10 da. at 5, 6, 7 per cent.
3. $65 for 2 yr. 5 mo. 16 da. at 4, 7, 8 per cent.
4. $750 for 3 yr. 8 mo. 12 da. at 6, 8, 9 per cent.
5. $68.40 for 2 yr. 6 mo. 24 da. at 5, 7, 10 per cent.
6. $20 for 2 yr. 9 mo. 18 da. at 7, 9, 11 per cent.
7. What is the amount of $630.50 for 3 yr. 6 mo. at 10%?
8. What is the amount of $250 for 4 yr. 9 mo. 15 da. at 7%? 8%? 9%? 10%? 12%?
9. What is the interest and amount of $95 for 3 yr. 6 mo. 6 da. at 6%? 5%? 7%? 8%? 10%?
10. What is the interest of $840 from Dec. 25, 1894 to May 20, 1896, at 7%? Amount?
11. What is the interest of $1,260 at 5% from Jan. 10, 1892 to July 25, 1894?
12. B owes three sums of money, viz:

A note for $1,000 dated July 20, 1892;
A note for $1,500 dated Aug. 8, 1891;
A note for $2,000 dated Jan. 1, 1893;

if money is worth 8%, how much money will pay these notes Dec. 1, 1895?

Notes, Partial Payments and Bank Discount.

138. A **promissory note** is a written promise to pay a sum of money at a specified time to a certain person or his order.

139. The **face** of a note is the sum named in it to be paid.

140. The **maker** or **drawer** of a note is the party who signs the note. The **payee** is the party to whom the money is to be paid.

141. A note is **negotiable** when payable to "bearer" or to "order."

142. An **indorser** is one who becomes responsible for the payment of a note by writing his name on the back.

143. The **maturity** of a note is when it is legally due, including the three days of grace.

144. A **demand note** is due on presentation; a **time note** matures three days after the time specified in the note.

145. **Days of grace** are three days allowed by law for the payment of a note after it becomes payable.

146. Interest computed on the face of a note (or amount when the note is on interest) is called **bank discount.**

147. **Partial Payments** are payments which are made on notes or other obligations.

148. An **indorsement** is a statement of the amount and time of payment, generally written on the back of an obligation.

149. Obligations on which there have been partial payments are usually settled under the U. S. rule, which is based on the principle that the sum at interest shall not be increased by adding the interest when greater than the payment.

150. 1. The following note was indorsed April 18, 1883, $65; Oct. 30, 1884, $40; Feb. 14, 1885, $200; how much was due May 9, 1885?

2. $500. St. Louis, Mo., Jan. 12, 1883.

Thirty days after date, for value received, I promise to pay Wm. Jones or order Five Hundred Dollars, with interest at 6% per annum.

JAMES SMITH.

yr.	mo.	da.	yr.	mo.	da.
1885	5	9			
1885	2	14		2	25
1884	10	30		3	14
1883	4	18	1	6	12
1883	1	12		3	6
2	3	27=	2	3	27

Principal,	$500.00
Int. for 3 mo. 6 da.,	5.50
Amount due,	505.50
1st payment,	65.00
New principal,	440.50
Int. 1 yr. 6mo. 12 da.,	40.53
Int. 3 mo. 14 da.	7.50
Amount due,	488.53
2d and 3d payments,	240.00
New principal,	248.53
Int. 2 mo. 25 da.	3.52
Amount due,	$252.05

Solution.—Arranging the dates in their order and subtracting the lowest, date of the note, from the date of the first payment, we obtain 3 mo. and 6 da., the time for which interest is to be computed on the principal; subtracting the date of the first payment from the date of the second payment we obtain 1 yr. 6 mo. and 12 da. the time for which interest is to be computed, on the remaining amount due; continuing to subtract each date from the one directly above it we obtain the several periods for which interest must be computed on the sum remaining unpaid. The interest from April 18, 1883, to

October 30, 1884, exceeding the amount paid at that time is not added to the principal until after the interest for the next period is computed when, the sum of the two payments exceeding the two amounts of interest due, the interest is added and both payments subtracted. See § 149.

When the interest for each period has been added to the principal still due and the payments subtracted, the interest is then computed on the remaining amount due until the time of final settlement.

Note.—By carrying the difference between the several dates to the right and comparing their sum with the difference between the earliest date and the latest date, any mistake in subtraction may be detected.

3. How much is due Dec. 1, 1894, on a note for $600, dated Jan. 10, 1892, bearing 7%, on which the following payments have been made: Jan. 1, 1893, $100; Aug. 10, 1894, $10?

4. What will be due Oct. 1, 1895 on a debt of $1,200, dated Jan. 10, 1893, bearing 8% interest, on which $200 was paid Jan. 12, 1894; $500 June 20, 1894; $200 Jan. 12, 1895?

151. 1. A note of $400, dated June 10, 1863, for 4 mo. was discounted at the bank at 6%, on Aug. 10, 1893; find date of maturity, term of discount, and proceeds.

$400
.06
——
12)$24.00. Int. for 1 yr.
——
30)2.00. Int. for 1 mo.
——
.06666. Int. for 1 da.
64
——
$4.26624. Int. or Disc. for 64 da.
$400—$4.27=$395.73=Proceeds.

June 10+4 mo. 3 da.
=Oct. 13, date of mat.
Days in Aug. after 10, 21
Days in Sept. 30
Days in Oct. to 13, 13
——
Total days for discount, 64

Problems.

2. What will be the discount and proceeds of a note for $68 dated June 20, 1892, for 90 days, if discounted at the bank at 6%, July 1, 1892?

3. A note for $800 dated June 25, 1891, for 60 days, was discounted on date of note at 10%; find the discount and proceeds.

4. A note for $60 dated May 1, 1890, for 6 mo. at 6% interest, was discounted Oct. 30 at 9%; find the proceeds.

Suggestion.—Find the amount of note and use it as the face to discount.

Find the discount and proceeds on the following notes:

	Date.	Face.	Time.	Rate.	Date of Dis.	% of Dis.
5.	June 20, '92,	$600,	3 mo.		June 20, '92,	10%
6.	Dec. 15, '93,	$1200,	4 mo.	6%,	Jan. 20, '94,	6%
7.	Mar. 20, '94,	$60,	90 da.	9%,	April 1, '94,	12%

Find the day of maturity, term of discount, discount, and the proceeds on the following notes:

8. $650.50. New York, May 12, 1885.

Six months after date I promise to pay to the order of Thomas Day $650\frac{50}{100}$, for value received.

Discounted at 6%, June 1. John Howard.

9. $3,500. Springfield, Mass., Jan. 1, 1892.

Three months after date I promise to pay to the order of Charles W. Ray $3,500, with 6% interest, for value received.

Discounted at 7%, Jan. 14. Austin Hendrickson.

10. $534.50. St. Louis, Feb. 25, 1889.

Sixty days from date I promise to pay A. W. Birge, or order, $534\frac{50}{100}$, with int. at 6% per annum, for value received.

Discounted at $6\frac{1}{2}$%, Feb. 28. Henry Bradshaw.

11. A note of $2,000, dated March 22, 1886, drawing interest at 6%, had payments indorsed upon it as follows: May 20, 1886, $850; July 20, 1886, $162.50; Dec. 23, 1886, $75.50. Find the balance due May 12, 1887.

12. A note of $3,525, dated Jan. 14, 1887, drawing interest at 5%, had payments indorsed upon it as follows: June 12, 1887, $1,200; Jan. 30, 1888, $1,500. Find the balance due May 13, 1889.

13. Find the amount of $600 placed at interest at 6%, for 3 yr. 2 mo. 19 da.

14. Bought a horse at 20% less than the seller's asking price, and sold him for 20% more than that asking price, and gained $60; what was the cost and selling price?

15. Sold $\frac{1}{2}$ an article for 20% less than cost; at what per cent. of cost must I sell the remainder to gain 10% net on the whole article?

16. The tax in a certain district is 4 mills on the dollar for school purposes, 5 mills for State tax, and 12 mills for city government; what is the rate of tax on $1? What would be a man's tax if his property was assessed at $10,000?

17. If the assessed value of a city is $6,000,000, what rate of tax must be assessed to raise $120,000?

18. If a collector gets 2% on his collections, what sum must be collected to realize $6,860 net?

19. A merchant pays $7.20 per dozen for shovels; at what price should he sell each in order to make 25%?

20. Molasses cost $15 per hhd.; at what price per gallon must it be sold that the grocer may make 30%?

21. A speculator buys 640 acres of timber land for $9,600; for how much per acre must he sell it in order to realize $33\frac{1}{3}$% profit?

RATIO and PROPORTION.

152. **Ratio** is the comparison of like numbers, and is found by dividing the first number, or **Antecedent**, by the second number, or **Consequent**. Thus, the ratio of 16 to 8 is 2, or $16 \div 8 = 2$.

153. **Ratio** is an abstract number, and can be expressed only between numbers of like units. Thus, the ratio of 12 oz. to 3 oz. is 4. There can be no ratio of 12 oz. to 4 mi. Ratio corresponds to problem III under §104, and may be indicated by the sign of division; as a fraction; or by the colon (:), which is read, *is to*. The ratio of 12 men to 3 men may be expressed by any one of the following methods: 12 men÷3 men; $\frac{12 \text{ men}}{3 \text{ men}}$; or, 12 men : 3 men.

Principles of Ratio.

154. 1 Antecedent÷by consequent=ratio.

2. Antecedent÷by ratio=consequent.

3. Consequent×ratio=antecedent.

Problems.

1. What is the ratio of 36A. to 12A.? to 30A.? to 120A.?

2. What is the ratio of 3 bu. 2 pk. to 1 bu. 3 pk.? to 35 bu.?

3. The antecedent is 1 bu. 2 pk., the ratio 5; what is the consequent?

4. The consequent is 1 da. 8 hr. 20 min., the ratio 6; find the antecedent.

5. Find the ratio of 13 da. to 39 da.; of 3 mo. 10 da. to 10 mo.

6. Find the missing terms in the following, if the ratio is 5 in each case; (?) : 10 oz.; 3 bu. 2 pk. : (?); $6 : (?).

155. Two equal ratios may be connected by = or a double colon : : (*as*) and form a **Proportion.** Thus, 12 : 6=8 : 4; 8 : 2 : : 24 : 6; read 12 is to 6 as 8 is to 4; 8 is to 2 as 24 is to 6. Each ratio of a proportion forms a **Couplet,** and the antecedent of the first couplet and the consequent of the second couplet (1st and 4th terms) are the **Extremes;** the other two terms (2d and 3d) are the **Means.**

156. Write three ratios equal to the following three ratios: 12 bu. : 16 bu.; 8 men : 24 men; $16 : $20. Form proportions by connecting the equal ratios by : : or =. Observe that in each of these proportions the product of the extremes equals the product of the means. Hence to find a missing term;

Rule.—Divide the product of the extremes by the given mean to find the missing mean; or, divide the product of the means by the given extreme to find the missing extreme.

157. Find the missing term in the following proportions:

1. 36 : 18 : : (?):24.
2. 10 oz. : 30 oz. : : $60 : (?).
3. 60A. : (?) : : $75 : $175.
4. 7 men : (?) : : 60A. : 180A.
5. (?) : $18 : : 6 bu. : 24 bu.
6. 10 gal. : 15 gal. : : $90 : ?

158. A **Compound Ratio** is the ratio of the product of two or more numbers to the product of two or more like numbers having the same relation. Thus, the ratio of the surface of a board 16 in. long, 8 in. wide, to the surface of another board 10 in. long, 4 in. wide is—

$$16\times 8 : 10\times 4, \text{ or } \left.\begin{matrix}16\\8\end{matrix}\right\} : \left.\begin{matrix}10\\4\end{matrix}\right\}, \text{ or } \left.\begin{matrix}16 : 10\\8 : 4\end{matrix}\right\}$$

The ratio of 16 : 10 is $\frac{8}{5}$; of 8 : 4 is 2, and $\frac{8}{5}\times 2=\frac{16}{5}$, the same as $16\times 8 : 10\times 4$, or the true ratio.

159. A Compound Proportion is a proportion which contains one or more compound ratios.

Note.—When one factor of a term is missing the remaining factors of that term, together with the other extreme (or mean), are used as a divisor. Thus, in the proportion—

$$\left.\begin{matrix}16\\8\end{matrix}\right\} : \left.\begin{matrix}12\\(?)\end{matrix}\right\} :: 5 : 20, \qquad \frac{16\times8\times20}{12\times5}=42\tfrac{2}{3},\ \text{missing term.}$$

Exercises.

Find the missing term in the following:

1. $3\frac{1}{2} : \times :: 7 : 42$

2. $\left.\begin{matrix}2\frac{1}{2}\\5\end{matrix}\right\} : \left.\begin{matrix}\times\\6\end{matrix}\right\} :: 9 : 10\frac{4}{5}$

3. $\left.\begin{matrix}3\\6\\8\end{matrix}\right\} : \left.\begin{matrix}9\\2\\16\end{matrix}\right\} :: \left.\begin{matrix}8\\(?)\end{matrix}\right\} : \left.\begin{matrix}32\\4\end{matrix}\right\}$

4. $\left.\begin{matrix}7\frac{1}{2}\\3\end{matrix}\right\} : \left.\begin{matrix}4\frac{1}{3}\\9\end{matrix}\right\} :: \left.\begin{matrix}4\frac{3}{4}\\3\frac{1}{3}\end{matrix}\right\} :: \left.\begin{matrix}2\frac{3}{5}\\(?)\end{matrix}\right\}$

5. $\left.\begin{matrix}1\frac{1}{2}\\4\\3\frac{1}{5}\end{matrix}\right\} : \left.\begin{matrix}2\frac{1}{2}\\(?)\\2\frac{2}{3}\end{matrix}\right\} :: 175 : 600$

Problems.

1. If 6 horses eat 47 bu. of oats in a given time, how many bu. of oats will 36 horses require in the same time?

6 horses : 36 horses : : 47 bu. : X bu.
36 horses : 6 horses : : X bu. : 47 bu.
47 bu. : X bu. : : 6 horses : 36 horses.
X bu. : 47 bu. : : 36 horses : 6 horses.

$$\frac{36\times47}{6}= 282\ \text{bu. Ans.}$$

Analysis.—The ratio between the number of horses fed is the same as the ratio between the bushels consumed. Hence the ratio of the horses forms one couplet and the ratio of the bushels the other couplet of the proportion.

Note.—Observe that whichever ratio is used for the first couplet, the resulting equation is the same, so long as the third term is related to the first by the conditions of the problem.

2. How many acres will 10 men mow in a given time, if 4 men can mow 20 acres in the same time?

3. At a certain time a staff 6 ft. high casts a shadow 4 ft. long; how high is a church steeple that casts a shadow of 120 ft. at the same time?

4. If it requires 16 yd. to make 7 suits of clothes, how many suits can be made from 64 yards?

5. A man travels 60 miles in 4 days; how many days, traveling at the same rate, will he require to go 420 miles?

6. If a block of granite 12 in. long, 6 in. wide, 4 in. thick, weighs 50 lb., what will be the weight of a block of similar stone 8 in. long, 8 in. wide, and 6 in. thick?

$$\left.\begin{matrix}12\\6\\4\end{matrix}\right\}:\left.\begin{matrix}8\\8\\6\end{matrix}\right\}::50:X.$$

Note.—The ratio of the volumes must be the same as the ratio of the weights. Make three other proportions and observe that the resulting equation is the same.

7. If a block of lead, 10 in. long, 8 in. wide, 6 in. thick, weighs 150 lb., what is the width of a block of the same metal that is 12 in. long, 3 in. thick and weighs 300 lb.?

8. If it costs $60 to carpet a room 40 ft. by 30 ft., what is the width of a room 60 ft. long, which costs $80 for the same kind of carpet?

9. 7 men working 8 hours per day for 10 days can perform a certain amount of work; how many hours per day must 10 men work to perform 4 times as much work in 14 days?

10. If 26 bu. oats keep 6 horses, 11 days, how many bushels will keep 17 horses 24 days?

11. If a family using 3 burners 6 hours per day, when gas is $1.25 per 1,000 cu. ft., pay a bill of $4 per month, how much should a family using 6 burners, 8 hours per day, pay per month, if gas is $1.50 per 1,000 cu. ft.?

12. A and B hire a pasture of 60 acres at $10 per acre, agreeing that each shall pay in proportion to the amount of stock kept, one horse counting as two cows and two sheep as one cow. A has 10 horses in the pasture for 4 weeks, and 6 cows for 3 weeks; B has 12 cows in for 2 weeks, and 30 sheep for 2 weeks; how much should each pay towards the expense?

13. A garrison has provisions for 6,000 men for 12 weeks; after 6 weeks, 2,000 men are withdrawn; how long will the remaining provisions last the men who are left in the garrison?

14. A wheel which is 5.25 ft. in diameter measures 16.493475 ft. around it; what is the ratio of the circumference to the diameter?

15. There are provisions to support a garrison of 8,000 soldiers 60 days; how many soldiers must be withdrawn, so that the same provisions may last 80 days?

16. A man walks 24 miles in 8 hours. How many days will he require to walk 600 miles, walking 10 hours per day?

Properties of Numbers.

160. An **Even Number** is one that is divisible by 2; as, 4, 6, 22. An **Odd Number** is one that is not divisible by 2; as, 5, 7, 91.

161. A **Prime Number** is divisible only by itself and 1.

The prime numbers under 100 are 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

162. A **Composite Number** is the product of two or more prime numbers; as $16 = 2 \times 2 \times 2 \times 2$.

163. **Prime Factors** are the prime numbers which multiplied together produce composite numbers.

164. **Composite Numbers** may be resolved into prime factors whose product will equal the composite numbers; thus:

1. Find the prime factors of 252 and 924.

$$\begin{array}{rrl} 2)\underline{252} & 2)\underline{924} & \\ 2)\underline{126} & 2)\underline{462} & 252=2\times2\times3\times3\times7.\ \text{Ans.} \\ 3)\underline{63} & 3)\underline{231} & \\ 3)\underline{21} & 7)\underline{77} & 924=2\times2\times3\times7\times11.\ \text{Ans.} \\ 7 & 11 & \end{array}$$

2. Resolve 240; 90,552; 47,916 into their prime factors.

3. Resolve 14,828; 28,308 into their prime factors.

Note.—In finding prime factors, when all the smaller prime factors have been tried, in their numerical order, until a factor is reached which gives a quotient less than the trial divisor, the number can have no more exact factors.

165. The **Greatest Common Divisor** of two or more numbers, is the greatest number that will divide each of two or more numbers without a remainder.

166. The **Least Common Multiple** of two or more numbers, is the least number which can be divided, by each of two or more numbers, without a remainder.

167. 1. Find the **G. C. D.** and **L. C. M.** of 35, 70, and 420.

$$35=5\times7$$
$$70=2\times5\times7$$
$$420=2\times2\times3\times5\times7$$

Analysis.—It is evident the factors 5 and 7 being found in each and all of the three numbers, and no other factors being found in all of them, that their product, $5\times7=35$, must be the G. C. D.

A number to contain each of these three numbers must contain all of the factors of each of them; hence the product of $2\times 2\times 3\times 5\times 7=420$ must be the L. C. M.

Rule.—Separate the numbers into their prime factors; for the G. C. D. take the product of the factors which are found in each and all of the numbers, each factor taken the *least* number of times it is found in any number; for the L. C. M. take the product of all the different factors repeated the *greatest* number of times they appear in any number.

Find the G. C. D. and the L. C. M. of—

2. 72, 180, 108, 252.

3. 90, 126, 54, 144.

4. 25, 60, 100, 125.

5. 84, 100, 224, 300.

6. Find the L. C. M. of 54, 72, 112, 128.

7. Find the G. C. D. of 105, 140,245, 385.

168. 1. Find the sum of $\frac{13}{24}$, $\frac{11}{30}$, $\frac{17}{45}$.

$\frac{13}{24}=\frac{195}{360}$
$\frac{11}{30}=\frac{132}{360}$
$\frac{17}{45}=\frac{136}{360}$

$\frac{463}{360}=1\frac{103}{360}$. Ans.

$24=2\times 2\times 2\times 3$
$30=2\times 3\times 5$
$45=3\times 3\times 5$
L. C. M.$=2\times 2\times 2\times 3\times 3\times 5=360$.

Analysis.—By factoring, we find the L. C. M. of the denominators of the fractions; this will be the least common denominator of the fractions. By multiplying both terms of the fractions $\frac{13}{24}$ by the product of 3×5, the factors not found in its own denominator, we have the equivalent fraction $\frac{195}{360}$; by multiplying both terms of $\frac{11}{30}$ by $2\times 2\times 3$ we have the equivalent fraction $\frac{132}{360}$; and by multiplying both terms of $\frac{17}{45}$ by $2\times 2\times 2$ we have the equivalent fraction $\frac{136}{360}$. The sum of the three equivalent fractions is $\frac{463}{360}$,or $1\frac{103}{360}$.

Note.—This method of finding the least common denominator is convenient when the common denominator cannot be found by inspection. See p. 78, § 49.

Find the sum of—

2. $3\frac{13}{15}$, $4\frac{17}{18}$, $5\frac{11}{24}$,

3. $7\frac{11}{20}$, $3\frac{13}{21}$, $2\frac{17}{28}$, $5\frac{13}{35}$.

4. $2\frac{23}{30}$, $3\frac{11}{18}$, $2\frac{12}{35}$, $\frac{19}{90}$.

5. $1\frac{11}{27}$, $3\frac{13}{45}$, $6\frac{19}{30}$, $3\frac{17}{54}$.

6. Find the value of $8\frac{19}{24}+4\frac{13}{15}-(4\frac{7}{9}+1\frac{11}{20})$.

$$\begin{array}{lcl} 8\frac{19}{24}=8\frac{285}{360} & & 4\frac{7}{9}\ =4\frac{280}{360} \\ 4\frac{13}{15}=4\frac{312}{360} & & 1\frac{11}{20}=1\frac{198}{360} \\ \hline 12\frac{597}{360} & - & 5\frac{478}{360}=7\frac{119}{360}. \text{ Ans.} \end{array}$$

Suggestions.—Find the least common denominator of all the fractions and reduce to mixed numbers at the last.

7. Find the value of $8\frac{13}{24}+3\frac{11}{20}+2\frac{7}{16}-(3\frac{17}{18}+2\frac{37}{40})$.

8. From the sum of $7\frac{11}{16}$ and $9\frac{11}{18}$, take away the difference between $11\frac{13}{24}$ and $5\frac{37}{54}$.

Review Problems.

1. Find the cost of a field 80 rd. long, 55 rd. wide at $150 per acre. Find the gain if sold in lots 10 rd. by 5 rd. at $90 each. What is the per cent. of gain?

2. Find the cost of carpeting a room 15 ft. by 20 ft. at $1.60 per yd., carpet 54 in. wide.

3. Find the per cent. of gain on oranges bought at $2.25 per 100 when sold at 40 cents per doz.

4. What is the per cent. of gain on coal bought at $2 per ton when sold at 10 cents per bu. of 80 lb.?

5. Find the interest of $65 at 6% for 2 yr. 3 mo. 18 da.

6. A note for $90 is discounted at the bank at 8% for 93 days (including 3 days of grace); find the proceeds.

7. Find the income on six $1,000 bonds which bear $3\frac{65}{100}$%.

8. Find an agent's commission for selling a house for $6,500 at $2\frac{1}{2}$% commission. What does the owner receive net?

9. A sells a horse to B at a gain of 20%, and B sells him to C for $216, and loses 10%; what did A pay for the horse? What was B's loss?

10. The sum of two numbers is 360, and the less number is 114; what is the other number?

11. A merchant sold several pieces of cloth measuring respectively $16\frac{2}{3}$ yd., $14\frac{5}{8}$ yd., $18\frac{8}{9}$ yd., $17\frac{7}{12}$ yd., and $19\frac{3}{4}$ yd., at $\$3\frac{3}{8}$ a yard; how much did he receive for the five pieces of cloth?

12. A quantity of lumber is bought for $\$1363\frac{2}{3}$ and is sold for $\$1463\frac{3}{8}$; how much is gained by the transaction?

13. A farmer having 208 acres of land sold $92\frac{4}{5}$ acres; what is the value of what remained at $\$29\frac{7}{11}$ an acre?

14. A man gave his son $687\frac{4}{7}$ acres of land, which was $\frac{4}{7}$ of all the land he had; how much had he left?

15. Since there are 5280 feet in one mile, how many feet are there in $7\frac{3}{8}$ miles?

16. A sold 86,797 tons of coal in one year and B sold $\frac{7}{8}$ as much; how many tons did both sell?

17. Since there are $30\frac{1}{4}$ square yards in one square rod, how many square yards are there in 3827 square rods?

18. How many pairs of stockings at 31 cents a pair can be bought for 62 pounds of butter at 37 cents a pound?

19. A man having $8512 paid $4684 for a farm, $687 for a team, $876 for 26 cows and $356 for farming implements, how much money had he left?

20. A farm of 429 acres was bought at $74 an acre and sold for $97 an acre; how much was gained by the transaction?

21. A man earns $1364 a year and spends $891 a year; in how many years can he save enough to buy a house worth $4000?

22. A farmer gave $\$104\frac{7}{8}$ for a horse, and $\$97\frac{1}{2}$ for a yoke of oxen; he sold them both at a gain of $57; for how much did he sell them?

23. A cistern holding 140 gal. has a pipe by which 30 gal. will run into the cistern in 1 hour, and another that will discharge 20 gal. an hour; if both are running in what time will the cistern be filled?

24. A merchant bought $4\frac{3}{7}$ cords of wood at $\$3\frac{1}{4}$ per cord, and paid for it in cloth at $\$1\frac{4}{7}$ a yard; how many yards were required to pay for the wood?

25. If an army of 6000 men have provisions for 5 months, and 4500 men be discharged; how long will the same provisions last the remainder?

26. If 6 bbl. of flour cost $\$40\frac{1}{2}$, how much will $2\frac{2}{9}$ barrels cost?

27. Bought 728 lb. of candles at $16\frac{3}{8}$ ct. a lb.; had they been bought for $3\frac{7}{8}$ ct. less a lb.; how many lb. could have been bought for the same money?

28. A man bought some cows for $3,549 and sold them for $4,593, and thereby gained $9 on each cow; how many cows were there?

29. If $\frac{2}{3}$ of a ton of hay cost $\$8\frac{2}{3}$, how much will 5 tons cost?

30. A man owed me $4,786 and paid me only $2,374; I agreed to take the rest in flour at $9 per bbl.; how many barrels should I receive?

31. If $3\frac{2}{3}$ lb. of sugar cost 33 ct., how much will be paid for $65\frac{1}{2}$ lb.?

32. If $235\frac{1}{2}$ acres of land cost $\$4,725\frac{3}{8}$, how much will 628 acres cost?

33. If 14 acres of meadow land produce $32\frac{2}{3}$ tons of hay, how many tons will 5 acres produce?

34. I have 7 bags of grain; 6 of them contain 89 lb. each, they all contain 722 lb..; how many lb. does the 7th contain?

35. If the yearly salaries of the United States Ministers to Russia, Spain, Mexico and Brazil is $12,000 each, what is the amount of all their salaries for 4 years?

36. One man travels 67 miles a day, another man follows at the rate of 42 miles a day; if they both start from the same place at the same time, how far apart will they be at the end of 6 days?

37. A farmer sold an equal number of chickens, ducks and geese for $35.40; the chickens at 12 ct. each, the ducks at 37 ct. each and the geese at 69 ct. each; how many of each kind did he sell?

38. One man travels 75 miles a day, another man follows at the rate of 42 miles a day; if both started from the same place at the same time, how far apart will they be in 27 days?

39. At $\$\frac{2}{3}$ per yd., how many yards of muslin can be bought for $16?

40. At $\frac{4}{5}$ ct. each, how many oranges can I buy for $9\frac{3}{5}$ ct.?

41. If a man earns $\$8\frac{9}{10}$ in 1 week, how much will he earn in 12 weeks?

42. There are 14 fields in a farm; in the first four fields there are 153 acres, in eight other fields 296 acres and in the remaining fields 53 acres. What is the average number of acres in each field?

43. If a gill of oil cost 3 cents, how much will a gallon cost?

44. A man bought a farm of 45,375 acres for $7,269; how much did it cost him per acre?

45. If I buy 2 bu. of walnuts at $1.50 a bushel and sell them at 3 ct. a pint, how much do I gain?

46. I bought 27 horses for $1,400, 3 died; for how much each must I sell the remainder to incur no loss?

47. I bought flour at $1.90 a sack and sold it for $919.05 thereby losing on each sack 25 cts. How many sacks of flour did I buy?

48. I bought 38 pieces of silk containing 42 yards each, at $1.75 per yard. Find the cost.

49. I bought cattle for $585 and sold them for $666 thereby gaining $9 on each head; how many cattle did I buy?

50. How many pupils in 5 schools, each school having 12 rooms, with an average of 53 pupils in each room?

51. There are 14 bins, each containing 276 bu. of wheat; what is it all worth at 115 ct. a bu.?

52. How many cattle in 3 trains of 26 cars, with 15 head in each car?

53. A farm of 19 fields has 27 acres in each field. What is the farm worth at $36 an acre?

54. I bought 325 loads of wheat, each load containing 50 bu., at $2 a bu.; what did the wheat cost?

55. In a building there are 72 rooms; in each room 4 windows and in each window 24 lights; how many lights in the house?

56. I spend 99 cents a day; how many cents will I spend in 49 years of 365 days each?

57. There is a book which contains 999 pages and each page averages 346 words, and each word averages 7 letters; how many letters in the book?

58. There is a pile of lumber containing 47 boards; what is it worth at 3 ct. per foot, if each board contains 19 bd.ft.?

59. A merchant gave 26 boxes of lemons for 7 firkins of butter worth $12 per firkin; what was the value of the lemons per box?

60. A merchant bought 39 firkins of butter containing 112 lb. each at 38 ct. per lb. How much did the butter cost?

61. Change .675 A. to integers of lower denominations.

62. Find the cost of building the walls of a house 40 ft. long, 18 ft. wide, 30 ft. high, if the walls are 13 in. thick and the brick cost $11 per 1,000.

63. A cistern contains 90 hhd. of water, and has two pipes, by one of which 6 gallons of water per minute run in, and by the other 11 gallons run out. In how many hours will the cistern be empty if both faucets are open at the same time?

64. What per cent. is gained when $\frac{3}{5}$ of an article is sold for what $\frac{7}{8}$ of it cost?

65. Find the entire surface of a block of wood 2 ft. square and 6 ft. long.

66. How many feet of boards 1 in. thick can be sawed from the block in example 65, allowing nothing for waste?

67. From a cask of wine containing 40 gal., 8 gal. leaked out. What per cent. of the wine remained? If the wine cost 80c. per gal., at what price per qt. must the remainder be sold to gain 25% on the entire cost?

68. How many yd. of lining $\frac{3}{4}$ yd. wide must be used to line 60 garments, each containing $4\frac{1}{2}$ yd. of material 1 yd. wide? (State by proportion.)

69. How long after a flash of lightning from a cloud $1\frac{1}{2}$ mi. away will the thunder be heard, if sound travels 1,130 ft. per second?

70. How many days from the fourth of July to Christmas?

71. A cistern has a capacity of 100 bbl. of water; how many bu. of grain would it contain?

72. A grocer buys flour at $4 per bbl. and sells it at $4.25; what per cent. does he gain?

73. What is the per cent. of loss in selling tea at 50 cents that cost 60 cents?

74. Which is the better investment, land at $80 per acre that rents for $10 an acre or 6% bonds at par, if the expenses on the land are $2 per acre each year? What per cent. better?

75. How many 4% bonds of $100 each must I purchase to have an income of $800? What will they cost at 120?

76. My income this year is 25% more than last year; I have spent 60% of my income this year and saved $800; what was my income last year?

77. A drover bought 840 sheep at $4 a head; if 75 of the sheep die on the way to market, and the remainder are sold at the rate of $16 for three, what per cent will be made on the entire cost?

78. I sent my agent a car load of oats to be sold on commission and the proceeds invested in pork at 10c. per lb. If my agent charges $1\frac{1}{2}$% for buying, and 2% for selling, and sends me 68,600 lb. of pork, how many bu. of oats were in the car, if they were sold at 40c. per bu.?

79. B bought a farm of 397.8 A. at $75 an acre and sold it so as to gain 14%; how much did he make by the transaction?

80. How much will I gain by buying 250 feet of ground at $21 a foot, and selling it for $37 a foot, after holding it 5 yr. 9 mo. and 21 da., money being worth 8% per annum?

81. An agent sold 19,475 bbl. of flour at $4.75 per bbl.; what was his commission at 2% of the gross sales?

82. A wholesale grocer sold $100,000's worth of groceries at a profit of 15%; how much did he gain?

83. Find the interest on $891.78 for 3 yr. 7 mo. 18 da. at 7% per annum?

84. If a man borrows $430 for 3 yr. 7 mo. 27 da. at 8%; how much will he owe the lender at the end of that time?

ANSWERS TO PROBLEMS.

☞As the Exercises in this book are for drill purposes and are to be used in class to fix and test the pupil's knowledge of the processes, no answers to them are given.

Pages 27, 28, 29 and 30.—1. 2,274 sheep; **2.** 68,109 A.; **3.** $1,485,484; **4.** 3,597 bbl.; **5.** $4,731; **6.** 9 birds; **7.** 10 mar.; **8.** $18; **9.** 10 bk.; **10.** 9 bu.; **11.** 11 yd.; **12.** 8 wk.; **13.** 8 ap.; **14.** 16 pt.; **15.** $4; **16.** 10 pc.; **17.** 12 bys.; **18.** 9 mi.; **19.** 7 bu.; **20.** 12 bbl.; **21.** 8 rabs.; **22.** 14 pch.; **23.** $10; **24.** 10 mar.; **25.** 12 mar.; **26.** $14. **27.** 12 ap.; **28.** 11 gs.; **29.** 10 rg.; **30.** 9 Ap.: **31.** $8; **32.** 16 chi.; **33.** $18; **34.** 5 b.

Pages 32 and 33.—1. 13 ct; **2.** 6 mar.; **3.** 27 ct.; 18 ct.; 9 ct.; **4.** 8 tops; 12; **5.** 9 Ap.

Pages 35 and 36.—1. $137.77; **2.** $3,239.71; **3.** $7,150.38; **4.** $25,840; **5.** 2,062⅓ bu.; **6.** $328.79; **7.** 4 bbl.; **8.** $7; **9.** $6; **10.** 21 tr.; **11.** 18 ft.; **12.** 8 bats; **13.** 12 lb.; **14.** $7; **15.** 6 ps.; **16.** $5; **17.** 7 wk; **18.** 19 bars; **19.** 7 da.; **20.** 15 ct.; **21.** $4; **22.** $4; **23.** 13 sheep; **24.** 27 ct.; **25.** 3 bu.; **26.** 4 h.; **27.** 15 ct.

Page 37.—28. 7 Ap.; **29.** $6; **30.** $7; **31.** 18 ct.; **32.** 18 ct.; **33.** 9 boxes; **34.** 9 mi.; **35.** $24; **36.** 8 per.; **37.** 128 A.; **38.** $224; **39.** 1,783 oxen; **40.** $13,728; **41.** 523 T.

Pages 39, 40 and 41.—1. 31 ct.; **2.** 4 ct.; **3.** 28 ct.; 21 ct.; 14 ct.; 7 ct.; **4.** 8 l.; **5.** 9 pch.; **6.** 6 lb.; **7.** 19 mar.; **8.** 8 pins; **9.** 28 ct.; **10.** $6; **11.** $7; **12.** 13 doz.; **13.** 16 ct.; **14.** 8 pa. **15.** 1,358 bu.; **16.** 9 chinas; **17.** $2; **18.** $4; **19.** 8 da.; **20.** 36 ct.; **21.** 2,413 bbl.; **22.** 180 lb.; **23.** 129 A.; **24.** $968; **25.** 1,567 br.; **26.** $6,767; **27.** 1,786 pp.; **28.** 2,220 men; **29.** $3,906; **30.** $86; **31.** $2,756; **32.** 4,840 hills; **33.** 31 hr.; **34.** 1,952 qt.; **35.** 209 sheep; **36.** 28 ct.; **37.** 3 bu.; **38.** $249,967.73; **39.** $50,907.79; **40.** $2,969,638; **41.** 2,431 hogs; **42.** $19,107.08.

Pages 43, 44, 45 and 46.—1. 56,522 men; **2.** $90,939.417.98; **3.** $3,633,880; **4.** 152,695 sheep; **5.** $\$1,852,492\frac{1}{5}$; **6.** $\$1,504\frac{4}{5}$; **7.** 4,999 men; **8.** 2,869 bu.; **9.** $240; **10.** $25,410; **11.** $22; **12.** $60; **13.** $9; **14.** 686 men; **15.** $6,313; **16.** 186 cars; **17.** 858 A.; **18.** $300; **19.** 4 or.; **20.** $40; **21.** 25 ct.; **22.** 1,276 cd.; **23.** 488 A.; **24.** 29 teams; **25.** 3,204 yd.; **26.** 276 A.; **27.** 10 ct.; **28.** $19; **29.** 9 wag.; **30.** 1,354 A.; **31.** 4,336 ft.; **32.** 1,990 sheep; **33.** 301 lem.; **34.** 805 chil.; **35.** $6; **36.** $5; **37.** 8 yr.; **38.** 70 sheep; **39.** $849; **40.** 285 yd.

Pages 47, 48, 49 and 50.—1. 33 ct.; **2.** 47 pupils; **3.** 72 ct.; **4.** 9 bbl.; **5.** $8; **6.** 3,249 yd.; **7.** $39,336; **8.** $2,813; **9.** 1,636 bk.; **10.** $72; **11.** 16 ht.; **12.** 63 ct.; **13.** $4,592; **14.** 1,593 yd.; **15.** 1.859 men; **16.** $1,948; **17.** $12,136; **18.** $33; **19.** 14 yd.; **20.** $35; **21.** 520 bu.; **22.** $147,742; **23.** 62 da.; **24.** $115,444; **25.** 1,830 sheep; **26.** 8 rd.; **27.** 85 cans; **28.** $54; **29.** $153,960\frac{4}{6}$ bu.; **30.** $3,921,264; **31.** 1,904 mi.; **32.** 131,311 bbl.; **33.** $910,712; **34.** $42; **35.** 12 per.; **36.** 40 bu.; **37.** 1,118 bbl.; **38.** 98 rd.; **39.** 101 sets; **40.** $3,584; **41.** $441\frac{1}{3}$ lb.; **42.** 12 ct.; **43.** 188 mi.; **44.** 883 yd.; **45.** 1,126 bu.

Pages 52, 53 and 54.—1. 42 ct.; **2.** 16 ap.; **3.** 1,073,841 ti.; **4.** $949,119; **5.** 12,795 ft.; **6.** 2,024,327 bu.; **7.** 6,772,923 sh.; **8.** 89 sheep; **9.** 63 ct.; **10.** 13 bk.; **11.** 2,225,610 hills; **12.** 105,311 A.; **13.** $253; **14.** 3,106 spoons; **15.** $49,959; **16.** $15; **17.** 12 ap.; **18.** $5.25; **19.** $279,601; **20.** 103 pas.; **21.** 10,464 lb.; **22.** 1,799 bu.; **23.** 64,780 votes; **24.** $84; **25.** 30 ct.; **26.** 4 sheep; **27.** 10,391 sh.; **28.** 28,372 sheep; **29.** 5,150,082 br.; **30.** $538; **31.** 147 mi.; **32.** $1.20; **33.** $1,594; **34.** 318 ct.; **35.** 1,019 mar.; **36.** 11 tr.; **37.** 63 da.; **38.** 28 ct.; **39.** 5,123 coats; **40.** 2,975 mi.; **41.** 112 turk.; **42.** $107,689.

Pages 56 and 57.—1. 12 sheep; **2.** 6 ct.; **3.** 23 pch.; **4.** 304 mi.; **5.** 520 rings; **6.** 349,043 bk.; **7.** 3,373,312 bu.; **8.** 7,497 sheep; **9.** 26 ct.; **10.** 11 papers; **11.** $72; **12.** 9,744 pen.; **13.** 439 letters; **14.** 688 ft.; **15.** 102 lb.; **16.** 5,934 ct.; **17.** 12 bu.; **18.** 83 bbl.; **19.** 45 ct.; **20.** $2,544; **21.** 4,540 seats; **22.** 10,336 pict.; **23.** 52,830 men; **24.** 1,504 ears; **25.** 8 fail.; **26.** $39; **27.** $27; **28.** 665 children.

Pages 59, 60 and 61.—1. 850,450 lb.; **2.** $19,066; **3.** $2,601; **4.** 9 pairs; **5.** $32; **6.** $81; **7.** $37,962; **8.** 197 bbl.; **9.** 2,562 cd.; **10.** 1,121 per.; **11.** 1,306 bbl.; **12.** 11 tp.; **13.** 29 ex.; **14.** $819; **15.** 857 lb.; **16.** 906 men; **17.** 376 boys; **18.** 261 mi.; **19.** 5,844 pp.; **20.** 8 ct.; **21.** 25 pupils; **22.** $56; **23.** 321 cans; **24.** $1,612; **25.** 1,183 A.; **26.** 1,296 spools; **27.** 202 chi.; **28.** $7; **29.** $63; **30.** $125; **31.** $7,101; **32.** $\$2,163.09\frac{8}{9}$; **33.** 1,681 bbl.; **34.** $7,172.37; **35.** $37,892,196.25; **36.** 243 mi.; **37.** 196 hr.; **38.** 5 mi. an hr.; **39.** $1,183 gain; **40.** 768 bbl.; **41.** 26 hr.

Page 65.—1. $53.79; $58.68; **2.** 722,610 slates; 895,681 slates; **3.** 613,029 mel.; 728,857; **4.** 80,206,203 ft.; **5.** 8,460,972 sq. yd.; **6.** 715 oz.; **7.** 63,740 lb.; **8.** $157,752.

Page 66.—1. $32,523; **2.** $34,868; **3.** $572.30; **4.** 5,073 mi.; **5.** 197,532 in.; **6.** 31,296 qt.

Pages 69, 70 and 71.—1. 949 bu.; **2.** 83 ct.; **3.** 23 mi. an hr.; **4.** 29 da.; **5.** 110 hhd.; **6.** $537\frac{2}{3}\frac{8}{6}$ yd.; **7.** 97 bu.; **8.** 230 bu.; **9.** 251 bu.; **10.** 1,000 ld.; **11.** $1,964 gain; **12.** $5,203\frac{2}{7}$ gal.;

13. 21,859$\frac{13}{36}$ yd.; 14. 140 A.; 16. \$2,892; 17. \$615,875; 18. 5,742 men; 19. \$8,512; 20. \$2,524; 21. \$1,006.11; 22. 918 sheep.

Page 71.—I.—2. 10,640; 3. 900,998,998; 4. 96,771,637,326; 5. 7,573,133$\frac{86}{89}$.—II.—1. 890,989,209; 2. 10,642,386,458; 3. 56,123,917$\frac{5}{12}$; 4. 5,363,893,573; 5. 6,185,575$\frac{15}{97}$.

Page 72.—III.—2. 909,989,859; 3. 9,179,849,556; 4. 61,863,581$\frac{9}{12}$; 5. 10,352,883$\frac{26}{87}$.—IV.—1. 1,086,408,125; 2. 700,989,889; 3. 1,820,401$\frac{12}{95}$; 4. 68,598,502,986; 5. 10,521,269$\frac{49}{86}$.—V.—1. 3,039; 2. 9,829; 3. 672,779,479; 4. 9,036,007$\frac{9}{12}$; 5. 79,711,932$\frac{8}{12}$.—VI.—2. 30,739,725$\frac{9}{11}$; 3. 11,038; 4. 759,483,669; 5. 1,336,474$\frac{35}{739}$.

Page 77.—1. $\frac{11}{4}$; $\frac{34}{5}$; $\frac{47}{6}$; $\frac{89}{10}$; $\frac{106}{11}$; $\frac{74}{7}$; $\frac{77}{8}$; $\frac{97}{11}$; $\frac{79}{8}$; $\frac{20}{3}$; 2. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{7}{8}$; $\frac{7}{8}$; $\frac{7}{8}$; $\frac{6}{7}$; $\frac{4}{5}$; $\frac{3}{7}$; $\frac{2}{3}$.

Page 79.—1. 3$\frac{3}{8}$; 2. 3$\frac{5}{9}$; 3. 3$\frac{40}{63}$; 4. 3$\frac{3}{22}$; 5. 3$\frac{59}{72}$; 6. 3$\frac{2}{15}$; 7. 4$\frac{1}{72}$; 8. 3$\frac{107}{140}$; 9. 3$\frac{167}{252}$; 10. 5$\frac{1}{24}$.

Pages 79 and 80.—1. 3$\frac{59}{72}$; 2. 36; 3. 308$\frac{79}{105}$; 4. 810$\frac{13}{15}$; 5. 1,531$\frac{11}{18}$; 6. 21,349$\frac{37}{40}$; 7. 13,770$\frac{31}{36}$; 8. 304,812$\frac{35}{36}$; 9. 106,685$\frac{19}{120}$; 10. 101,100$\frac{31}{72}$.

Pages 80 and 81.—1. \$3$\frac{5}{24}$; 2. 532$\frac{1}{2}$ lb.; 3. 4$\frac{13}{18}$ yd.; 4. 34$\frac{1}{24}$ ft.; 5. 11$\frac{2}{3}$ bu.; 6. 21$\frac{23}{84}$ yd; 7. \$17$\frac{1}{3}$; 8. 308 A.; 9. \$329$\frac{13}{18}$; 10. \$5,168$\frac{53}{132}$.

Page 81.—1. $\frac{1}{12}$; 2. $\frac{13}{88}$; 3. $\frac{1}{15}$; 4. $\frac{13}{36}$; 5. $\frac{30}{77}$; 6. $\frac{11}{36}$; 7. $\frac{21}{44}$; 8. $\frac{5}{24}$; 9. $\frac{1}{9}$; 10. $\frac{1}{60}$.

Page 82.—§58.—1. 8,005,555$\frac{1}{8}$; 2. 6,251,232$\frac{5}{11}$; 3. 82,947,332$\frac{1}{9}$; 4. 8,242,159$\frac{5}{12}$; 5. 69,417,761$\frac{8}{11}$.

Page 82.—§59.—1. 6,940,319$\frac{82}{99}$; 2. 8,692,397$\frac{29}{36}$; 3. 79,986,173$\frac{61}{72}$; 4. 90,930,800$\frac{7}{66}$; 5. 6,999,993$\frac{79}{132}$.

Pages 82 and 83.—1. \$$\frac{1}{6}$; 2. \$2$\frac{1}{9}$; 3. 1$\frac{11}{12}$ bu.; 4. 11$\frac{71}{198}$ bu.; 5. \$2$\frac{17}{77}$; 6. \$40$\frac{63}{88}$; 7. \$27$\frac{5}{18}$; 8. \$41$\frac{5}{36}$; 9. \$44$\frac{23}{35}$; 10. \$12$\frac{34}{99}$; 11. 69$\frac{31}{36}$; 12. \$13$\frac{5}{72}$; 13. \$4$\frac{31}{36}$; 14. 683$\frac{35}{36}$ A.; 15. \$$\frac{23}{72}$.

Page 84.—*a*. 4$\frac{10}{11}$; *b*. 6$\frac{2}{9}$; *c*. 3$\frac{1}{2}$; *d*. 2$\frac{4}{7}$; *e*. 5$\frac{1}{2}$; *f*. 4; *g*. 4$\frac{1}{11}$; *h*. 6$\frac{2}{3}$.

Page 85.—1. 790$\frac{1}{2}$; 2. 719$\frac{1}{9}$; 3. 371$\frac{3}{7}$; 4. 878$\frac{2}{5}$; 5. 208$\frac{2}{3}$; 6. 513$\frac{9}{11}$; 7. 239$\frac{7}{12}$; 8. 399$\frac{1}{9}$; 9. 71$\frac{2}{11}$; 10. 667$\frac{5}{8}$ 11. 705$\frac{4}{9}$; 12. 679$\frac{7}{10}$; 13. 617$\frac{1}{7}$; 14. 63,734$\frac{1}{6}$; 15. 481$\frac{1}{4}$.

Page 86.—1. $\frac{7}{9}$; $\frac{63}{88}$; $\frac{54}{77}$; $\frac{35}{72}$; 2. $\frac{22}{27}$; $\frac{18}{25}$; $\frac{7}{15}$; $\frac{55}{108}$.

Pages 87, 88 and 89.—1. \$3$\frac{1}{3}$; 2. \$7$\frac{1}{2}$; 3. \$11$\frac{11}{12}$; 4. 9$\frac{5}{8}$ ct.; 5. \$8$\frac{1}{10}$; 6. \$15$\frac{2}{3}$; 7. \$3$\frac{2}{3}$; 8. \$9$\frac{26}{35}$; 9. \$3$\frac{1}{3}$; 10. \$9.57$\frac{7}{8}$; 11. \$144$\frac{5}{6}$; 12. \$81.53$\frac{1}{6}$; 13. \$2,719$\frac{1}{12}$; 14. \$4,743$\frac{3}{8}$; 15. \$216$\frac{19}{72}$; 16. \$1,035$\frac{7}{12}$; 17. \$11$\frac{21}{44}$; 18. \$3$\frac{1}{12}$; 19. \$4$\frac{20}{77}$; 20. \$1.92$\frac{1}{6}$; 21. \$3.55; 22. \$159$\frac{9}{11}$; 23. \$219$\frac{1}{3}$; 24. \$2.43$\frac{6}{7}$; 25. \$99$\frac{5}{11}$; 26. \$9$\frac{25}{36}$; 27. 7$\frac{4}{15}$; 28. $\frac{5}{11}$ more; 29. \$$\frac{14}{81}$; 30. \$$\frac{1}{3}$; 31. \$$\frac{35}{132}$;

32. $\$\frac{49}{99}$; **33.** $\$\frac{12}{77}$; **34.** $\$\frac{403}{1584}$; **35.** $\$4\frac{5}{33}$; **36.** $\$223\frac{1}{22}$; **37.** $\$262\frac{2}{11}$; **38.** $\$390\frac{55}{72}$; **39.** $\$4,555\frac{49}{88}$; **40.** $\$168\frac{13}{28}$; **41.** $\$113\frac{13}{36}$; **42.** $34\frac{11}{48}$ mi.; **43.** $\$58\frac{41}{96}$; **44.** $\$2.70\frac{11}{14}$; **45.** $\$6,600\frac{107}{108}$; **46.** $841,942\frac{25}{72}$.

Pages 95 and 96.—1. $\$\frac{1}{117}$; **2.** $\$\frac{2}{77}$; **3.** $\$\frac{5}{11}$; **4.** $\$1\frac{4}{7}$; **5.** $\$1\frac{3}{7}$; **6.** $\$\frac{11}{54}$; **7.** $\$\frac{44}{63}$; **8.** $\$5\frac{107}{132}$; **9.** $\$\frac{6}{55}$; **10.** $\$9\frac{3}{7}$; **11.** $\$77\frac{1}{7}$; **12.** $\$10\frac{2}{7}$; **13.** $\$19\frac{3}{5}$ ct.; **14.** $\$\frac{24}{49}$; **15.** $\$1\frac{2}{5}$; **16.** $\$2\frac{1}{5}$; **17.** $\$1\frac{3}{7}$; **18.** $\$13\frac{41}{56}$; **19.** $446\frac{1}{5}$ A.; **20.** $36\frac{67}{84}$ yd.; **21.** 846 sheep; **22.** 1,176 pupils; **23.** $\$3\frac{13}{24}$; **24.** $11\frac{5}{21}$ bu.; **25.** $10\frac{4}{11}$ yd..

Pages 96, 97 and 98.—26. $12\frac{107}{552}$ hr.; **27.** $12\frac{3}{29}$ bu.; **28.** $4\frac{32}{869}$ mi. A.; $3\frac{939}{1441}$ mi. B.; **29.** $4\frac{1}{12}$ da.; **30.** $2\frac{538}{1127}$ yd.; **31.** $43\frac{35}{67}$ hr.; $75\frac{69}{106}$ mi.; **32.** 5 lb.; **33.** $68\frac{197}{2296}$ bu.; **34.** $15\frac{29}{33}$ bu.; **35.** $\$8,252\frac{133}{324}$; **36.** $\$83\frac{5}{11}$; **37.** $1,632.64; **38.** $2,905.97; **39.** 283 mi.; **40.** $105 overdrawn; **41.** $911.

Page 102.—1. 3,202.63 A.; **2.** $66,397.84; **3.** $6,577.98; **4.** $10,765.19; **5.** $5.66.

Page 104.—1. $603.5712; **2.** $24,936.04; **3.** $701.3304; **4.** $715.55; **5.** $21,502.52.

Page 105.—21. 811,931 r.; **22.** $31.50; **23.** $536.41; **24.** 95,816.25 lb.; **25.** 5,100 A.; **26.** 9,888 lb.

Page 106.—1. $24.95; **2.** 305 lb.; **3.** 25.87 da.; **4.** 984.62 bu.; **5.** 167.6 da.

Pages 107 and 108.—1. 7,042 bu.; **2.** $46.70; **3.** 27,020 men; **4.** $56.88; **5.** 2.94 yd.; **6.** 635 sheep, B; 1,058 sheep, C; **7.** 696 bu. wheat; 520 bu. corn; **8.** 2,463 mi.; **9.** $16,607; **10.** $426; **11.** $2,179,060; **12.** $529.96.

Page 109.—1. 160 oz.; 240 oz.; 320 oz.; **2.** 10 lb.; 20 lb.; $7\frac{1}{2}$ lb; **3.** $388\frac{1}{2}$ bu.; **4.** 89 bbl.; **5.** 8 T.; $1\frac{4}{5}$ T.; **6.** $20; **7.** $3; **8.** $\$1.32\frac{3}{16}$; **9.** $40.25877.

Page 110.—§87.—1. 144 oz.; 216 oz.; 360 oz.; **2.** 28,800 gr.; 9,600 gr.; 33,600 gr.; **3.** $3.60; **4.** $1.80.—**§88.—1.** $23\frac{1}{3}$ oz.; 40 oz.; 50 oz; **2.** 2,400 gr.; 420 gr.; 864℈; **3.** 3,360 gr.; **4.** 84℈; 85 gr.

Page 111.—§89.—1. 320 pt.; 1,260 pt; **2.** 16 bbl.; 8 bbl; **3.** $12.75.—**§90.—1.** 1,920 pt.; 2,880 pt.; **2.** 160 pk.; 40 bu.; **3.** 1,312 pt.; **4.** $81.60; **5.** $1,941.03; **6.** $10\frac{2}{7}$ bbl.

Page 113.—1. $24\frac{24}{365}$ mo.; $2\frac{2}{365}$ yr.; **2.** 52,560 hr.; 189,734,400 sec.; **3.** 76 da.; 1 hr.; **4.** 2,880 min.—**§93.—1.** $1\frac{23}{33}$ mi.; 11 mi.; **2.** 443,520 in.; 190,080 in.; 570,240 in.; **3.** 4,841.27 mi.; **4.** 17,600 yd.; 35,200 yd.

Page 114.—1. $8,167\frac{1}{2}$ sq. ft.; **2.** 96 sq. yd.; $144; **3.** 24 sq. yd.; $48; **4.** $7.20; **5.** $142\frac{2}{9}$ sq. yd.; **6.** 224 sq. yd.

Page 115.—1. 3,960 cu. ft.; **2.** 720 cu. in.; **3.** $100; **4.** $.890625; **5.** 405 cu. ft.; **6.** 40 yd.; **7.** $110; **8.** $132; **9.** 384 cu. ft.; 256 cu. ft.; 192 cu. ft.; **10.** 10 cu. yd.; **11.** 65 da.; **12.** $1,700; **13.** $11\frac{1}{4}$ cd.; **14.** 660 min.; **15.** 4 lb.; 15 lb.; **16.** $160.08.

Page 116.—§96.—1. 12,000 d.; 48,000 far.; **2.** 100s; £5; **3.** £900; £1,200; **4.** $3,766.80; $3,328.80.—**§98.—1.** $10; 1 E.; **2.** 10,000 ct.; 100,000 m.; **3.** 100 nickels; 400 nickels; **4.** $17.50; **5.** 600 d.; **6.** 65 m.; **7.** 9,065m.; **8.** 1,000 ct.; $10; **9.** 74 ct. + 9 m.; **10.** $15.64 + 3 m.

Page 117.—§99.—1. 11 bu. 2 pk. 1 qt.; **2.** 19 bu. 3 pk. 3 qt; 12 bu. 0 pk. 1 qt.; 13 bu. 1 pk. 6 qt.; 12 bu. 1 pk. 3 qt.; **3.** 57 bu. 2 pk. 5 qt.; **4.** 19 lb. 7℥ 2ʒ 1℈; **5.** 13 da. 13 hr. 1 min. 48 sec.; **6.** 13 da. 13 hr. 1 min. 48 sec.; **7.** 14 mi. 286 rd. 1 ft. 11 in.; **8.** 14 mi. 286 rd. 1 ft. 11 in.

Page 118.—1. 22 hr. 9 min. 45 sec; **2.** 6 yr. 8 mo. 24 da.; **3.** ; **4.** **5.** 2 yr. 7 mo. 23 da.

Page 119.—1. 2 bu. 1 pk. 2 qt. $\frac{5}{8}$ pt.; 1 bu. 3 pk. 7 qt. $1\frac{4}{7}$ pt.; 1 bu. 2 pk. 1 qt. $1\frac{2}{5}$ pt.; **2.** 13 cd.; 7 cd. ft.; $15\frac{7}{8}$ cu. ft.; **3.** 9 T. 1 hhd. 1 gal. $2\frac{21}{23}$ qt.; 7 T. 0 hhd. 24 gal. $1\frac{7}{10}$ qt; **4.** 65 bu. 0 pk. $4\frac{1}{15}$ qt.; 40 bu. 2 pk. $6\frac{13}{24}$ qt.—**§102.—2.** 30,400 gr.; **3.** 188 gi.; **4.** 28,032 gr.; **6.** 2 lb. 10 oz. 19 pwt. 4 gr.; **7.** $8\frac{1788}{5760}$ lb.; **8.** $\frac{811}{960}$ lb.; **9.** $5\frac{5}{18}$ lb.

Page 120.—10. $5\frac{5}{18}$ lb.; $5\frac{7}{8}$ gal.; $4\frac{13}{15}$ lb.; **11.** $5.27\frac{7}{9}$ lb.; 5.875 gal.; $4.8\frac{2}{3}$ lb.; **12.** $\frac{1}{8}$ mi.; .125 of a mi.; **13.** .098 + A.; **14.** $1\frac{647}{1008}$ hhd.; 1.641 + hhd.—**§103.—2.** 1,350 lb.

Pages 121 and 122.—2. 1,350 lb.; **3.** 126 sq. rd.; 2 sq. yd.; 3 sq. ft.; 112.32 sq. in.; **4.** $44.80; **5.** 4 da. 9 hr.; **7.** $\frac{1}{64}$ lb.; **8.** $\frac{1}{1260}$ of an E.; **9.** $\frac{13}{32}$ gal.; **10.** 2 sq. rd.; 3 sq. yd.; 3 sq. ft.; 36 sq. in.; **11.** .009765625 bu.; **12.** 3 A.; **13.** 44 spoons; **14.** $\frac{100}{363}$ A.; **15.** $$115.17\frac{3}{16}$; **16.** $12.90; **17.** $1,189.6875; **18.** $367.96875; **19.** 2 lb. 11 oz. 6 dr. $11\frac{7}{8}$ gr.; **20.** 2 lb. 4 oz. 17 pwt. 12 gr.; **21.** 2 lb. 11 oz. $4\frac{4}{875}$ dr.; **22.** 1 lb. 14 oz. $10\frac{2}{35}$ dr.; **23.** 5 lb. 3℥ 3ʒ 1℈ 4 gr.; **24.** 1 lb. 2 oz. 12 pwt.; **25.** 60 rings; **26.** $15\frac{1}{2}$ min.; **27.** $1,267\frac{1}{5}$ times; **28.** 8 da.; **29.** 320 squares; **30.** $17\frac{6}{7}$ spoons; **31.** 16 lots; **32.** 67 yr. 9 mo. 22 da.; **33.** 320 rd.; **34.** $9\frac{1}{3}$ ft.; **35.** $33\frac{1181}{1320}$ mi.; $2\frac{8449}{12672}$ mi.

Pages 124 and 125.—1. $\frac{1}{3}$; **2.** 14; **3.** $2\frac{1}{5}$; **4.** $2\frac{1}{10}$; **5.** $1\frac{9}{16}$; **6.** $\frac{3}{40}$ A.; **7.** 128 sq. rd.; **8.** $\frac{3}{32}$ cd.; **9.** $\frac{77}{576}$ cu. ft.; **10.** 720 cu. in.; **11.** $550; $1,100; **12.** $\frac{7}{12}$ lb. Troy.; $\frac{7}{16}$ lb. Avoir.; **13.** $150; **14.** $\frac{3}{4}$ bu.; **15.** $52.60; **16.** 7 cd. ft.; **17.** 441 gal.; **19.** $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{3}{8}$; **20.** $0.75; $$0.87\frac{1}{2}$; $$0.16\frac{2}{3}$; $$0.06\frac{1}{4}$; $0.04; **22.** $275; **23.** $231.25; **24.** $110.00; **25.** $900; **26.** $1,000.—**§105.—2.** $1,305.60; $1,088; **3.** $24.408; $27.12; **4.** $54.24; **5.** $8.8257; **6.** $$1,081.98\frac{3}{4}$; **7.** $20.475; **8.** $10.50.

Page 126.—1. $146.40; **2.** $$40.58\frac{1}{2}$.

Pages 127 and 128.—§107.—1. $34.10; **2.** $3.10; **3.** $$18.66\frac{2}{3}$.—**§108.—1.** $8.90; **2.** $58.42; **3.** $$8.16\frac{2}{3}$; **4.** $$94.29\frac{3}{4}$; **5.** $3.60; **6.** $4,560; **7.** $15.00; **8.** $11.35.—**§109.—2.** $$793.33\frac{1}{3}$.

Page 129.—2. $3,600.83; **3.** $49.70; **4.** $132.84; **2.** $26.40; **3.** $95.40.

Page 130.—**2.** $77.60; **3.** $9.72; **1.** 360 bu.; 3,351.27 gal.; **2.** 168.7 bu.; 216.9 bu.; **3.** $3.61; **4.** 102.85 bu.; **5.** 5 cd.; **6.** $165.44; **7.** $153.60.

Page 132.—**2.** 1,360; 2,550; 1,785; 2.72; **3.** $4\frac{4}{5}$ A.; **4.** 70,000 men; **5.** 120 A.; 480 A.; **7.** 20; 50; 125; 150; 500; **8.** 72; 108; 432; 504; 648; **9.** 5 gal.; 2 hhd.; 20 bu.; 280 lb.; 13.6 mi.; **10.** $32; $68; $76.80; $147.60; $600; **11.** 8 A.; 64 A.; 128 A.; 16 A.; 400 A.; **12.** 270 lb.; 39 T.; 210 oz.; 390 lb.; 260 T.; **13.** $403.20; 783 ft. 3.6 in.; 195 gal. 1 qt. $\frac{2}{5}$ pt.; 35 A. 112 sq. rd.; 327 bu. 2 pk. 3 qt. $\frac{2}{5}$ pt.

Page 133.—**2.** 2,000; 72,000; **3.** 1,200; 96; **4.** 2,800; 4,800; **5.** 80,000; 16,000; **7.** 400; **8.** 2,000; **9.** 40; **10.** 700; **11.** 9 gal.; **2.** $66\frac{2}{3}$%; **4.** $8\frac{1}{3}$%.

Pages 134 to 137.—**5.** $18\frac{3}{4}$%; **6.** 9%; **7.** 25%; **8.** $3\frac{1}{2}$%; **9.** $3\frac{3}{4}$%; **10.** $1\frac{243}{257}$%; **11.** 4%; **12.** 50%; **13.** $57\frac{1}{7}$%; **14.** $33\frac{1}{3}$%; **15.** $2\frac{1}{2}$%; **16.** 50%; **17.** $131.25; **18.** 0.82\frac{1}{2}$; **19.** $5.50; **20.** $21.25; **21.** $2,600; $600; **22.** $3.200; **23.** $64.00; **24.** $\frac{2}{3}$ T.; **25.** $4\frac{1}{6}$%; **26.** $22. **27.** $666.40; **28.** $90 com.; $5,910; **29.** $11,850; **30.** $8,955; $.89$\frac{11}{20}$ per bu.; **31.** $22\frac{2}{9}$%; **32.** $16; **33.** $100; $66\frac{2}{3}$%; **34.** $10; $14\frac{2}{7}$%; **35.** $1\frac{1}{2}$%; $3,940; **36.** $9,000; $900 com.; **37.** $50; **38.** $200 carriage; $240 horse; **39.** $10,000 wheat; $20,000 corn; **40.** $38\frac{8}{9}$%; **41.** $4800; 25%; **42.** $16\frac{2}{3}$%; **43.** $16,000; $12,000; $14,000; **44.** $13,680; **45.** $287.28; **46.** $600; **47.** $36; **48.** $90; **49.** $\frac{3}{5}$%; **50.** $\frac{2}{5}$%; 2%; **51.** 2%; **52.** $2.50; **53.** 40%; **54.** $28\frac{4}{7}$%; **55.** 140%; **56.** $171\frac{3}{7}$% of 7; 240% of 5; 7 is $58\frac{1}{3}$%; 5 is $41\frac{2}{3}$; **57.** $0.50 Joe; $0.40 Wm.; **58.** $18\frac{3}{4}$%; **59.** $33\frac{1}{3}$%; **60.** 6.66\frac{2}{3}$; **61.** $6\frac{2}{3}$%; **62.** 75%; **63.** $57\frac{1}{7}$%; **64.** $22\frac{8}{11}$%; **65.** 20%.

Pages 138 and 139.—**2.** $7,190; **3.** $255; **4.** $4,602.50 he rec'd; **5.** $3,625; **6.** I paid $8,442.50; **7.** $975 loss; **8.** 300 shares; $1,800 income; **9.** $480.

Page 139.—**1.** $360; **2.** $480; **3.** $6,400; **4.** $255.50; **5.** $450; **6.** $640; **1.** $12,000; $8,000; $6,000; $4,800; **2.** $9,615; $6.410; $4,807.50; $3,846; **2a.** $14,415; $9,610; $7,207.50; $5,766; **1.** $6\frac{2}{3}$%; 5%; $3\frac{1}{3}$%; **2.** 10%; $7\frac{1}{2}$%; $6\frac{2}{3}$%; 5%.

Page 140.—**1.** 120; 75; 60; 50; **2.** $1,128; **3.** $2,870; **4.** 120 shares; **5.** $13,750; **6.** $96; **7.** $301; **8.** $11,200; **9.** $18,321; **10.** $11,718; **11.** $69\frac{3}{25}$; **12.** $2,000; **13.** $1,600; **14.** $360; **15.** $6,562.50; **16.** $15,937.50.

Page 141.—**2.** 17.93\frac{3}{4}$; **3.** $23.40; **4.** 49.87\frac{1}{2}$; **5.** $47.50; **6.** 0.38\frac{1}{3}$; $0.46; 0.53\frac{2}{3}$; 0.61\frac{1}{3}$; 0.76\frac{2}{3}$; **7.** $159.16; **8.** $421.40; **9.** $682.50.

Page 142.—**2.** 134.05\frac{5}{9}$; 160.86\frac{2}{3}$; 187.67\frac{7}{9}$; **3.** 6.39\frac{8}{9}$; 11.19\frac{7}{9}$; 12.79\frac{7}{9}$; **4.** $166.50; $222; $249.75; **5.** 8.77\frac{4}{5}$; $12.2892; 17.55\frac{3}{5}$; **6.** $3.92; $5.04; $6.16; **7.** $851.175; **8.** 333.85\frac{9}{20}$; 345.83\frac{1}{3}$; 357.81\frac{1}{4}$; 369.79\frac{1}{6}$; $393.75; **9.** $20.045; $16.704; $23.385; $26.726; $33.408; **10.** $70.70; $910.70; **11.** 160.12\frac{1}{2}$; **12.** $5,753.444$\frac{1}{2}$.

Page 145.—3. $603.53; **4.** $490.22.

Pages 146 and 147.—2. $67.07; **3.** $786; **4.** $61.75; **5.** Dis. $15.50; Pro. $584.50; **6.** Dis. $17.96; Pro. $1,206.64; **7.** Dis. $1.66; Pro. $59.74; **8.** Nov. 15; 167 da.; Dis. $18.11; Pro. $632.39; **9.** Apr. 4; 81 da.; Dis. $55.98; Pro. $3,498.27; **10.** Apr. 29; 60 da.; Dis. $5.85; Pro. $534.26; **11.** $991.45; **12.** $1,035-.97; **13.** $715.90; **14.** Cost $120; S. P. $180; **15.** 140%; **16.** $2\frac{1}{10}$%; $210; **17.** 2%; **18.** $7,000; **19.** $0.75; **20.** $0.31; **21.** $20.

Page 148.—1. 3; $1\frac{1}{5}$; $\frac{3}{10}$; **2.** 2; $\frac{1}{10}$; **3.** 1 pk. 1 qt. $1\frac{1}{5}$ pt.; **4.** 8 da. 2 hr.; **5.** $\frac{1}{3}$; $\frac{1}{3}$; **6.** 3 lb. 2 oz.; 2 pk. 6 qt. $\frac{4}{5}$ pt.; $1.25.

Page 149.—1. $48; **2.** $180; **3.** 140 A.; **4.** 21 men; **5.** $4.50; **6.** $135.

Pages 150 to 152.—1. 21; **2.** $2\frac{1}{2}$; **3.** 8; **4.** $10\frac{5}{9}$; **5.** $9\frac{153}{175}$; **2.** 50 A.; **3.** 180 ft.; **4.** 28 suits; **5.** 28 da.; **6.** $66\frac{2}{3}$ lbs.; **7.** $26\frac{2}{3}$ in.; **8.** 26 ft. 8 in.; **9.** 16 hr.; **10.** $160\frac{8}{11}$ bu.; **11.** $12.80; **12.** B's, $213.16; A's, $386.84. **13.** 9 wks.; **14.** 3.1416; **15.** 2,000 men; **16.** 20 da.

Page 153.—2. 2^4x3x5; 2^3x3x7^3x11; 2^2x3^2x11^3; **3.** 2^2x11x337; 2^2x3x7x337.

Page 154.—2. 36 G. C. D.; 7,560 L. C. M; **3.** 18 G. C. D.; 15,120 L. C. M.; **4.** 5 G. C. D.; 1,500 L. C. M.; **5.** 4 G. C. D.; 16,800 L. C. M.; **6.** 24,192; **7.** 35.

Pages 155 to 161.—2. $14\frac{97}{360}$; **3.** $19\frac{31}{210}$; **4.** $9\frac{49}{450}$; **5.** $14\frac{29}{45}$; **7.** $7\frac{95}{144}$; **8.** $11\frac{191}{432}$; **1.** $3,795; 92%; **2.** $35.55\frac{5}{9}$; **3.** $48\frac{4}{27}$%; **4.** 25%; **5.** $8.97; **6.** $88.14; **7.** $219; **8.** $162.50; $6,337.50; **9.** A's c. $200; B's l. $24; **10.** 246; **11.** $296.06; **12.** $99\frac{17}{24}$; **13.** $3,414.11; **14.** $515\frac{12}{28}$ A.; **15.** $41,653\frac{1}{3}$ ft.; **16.** $162,744\frac{3}{8}$ tons; **17.** $115,766\frac{3}{4}$ sq. yd.; **18.** 74 pr.; **19.** $1,909; **20.** $9,867; **21.** $8\frac{216}{473}$ yr.; **22.** \$$259\frac{3}{8}$; **23.** 14 hr.; **24.** $9\frac{7}{44}$ cd.; **25.** 20 mo.; **26.** $15; **27.** $953\frac{17}{25}$ lb.; **28.** 116 cows; **29.** $65; **30.** 268 bbl.; **31.** \$$5.89\frac{1}{2}$; **32.** $12,601; **33.** $11\frac{2}{3}$ tons; **34.** 188 lb.; **35.** $192,000; **36.** 150 mi.; **37.** 30; **38.** 891 mi.; **39.** 24 yd.; **40.** 12 or.; **41.** \$$106\frac{4}{5}$; **42.** $35\frac{6}{7}$ A.; **43.** 96 ct.; **44.** $16\frac{12}{605}$ ct.; **45.** 84 ct.; **46.** \$$58\frac{1}{3}$; **47.** 557 sacks; **48.** $2,793; **49.** 9 head; **50.** 3,180 pupils; **51.** $4,443.60; **52.** 1,170 head; **53.** $18,468; **54.** $32,500; **55.** 6,912 lights; **56.** $17.706.15; **57.** 2,419,578 letters; **58.** $26.79; **59.** \$$3\frac{3}{13}$; **60.** $1,659.84; **61.** 108 sq. rd.; **62.** $803.88; **63.** 18.9 hr.; **64.** $45\frac{5}{6}$%; **65.** 56 sq. ft.; **66.** 288 sq. ft.; **67.** 80% left; $31\frac{1}{4}$ ct.; **68.** $\left.\begin{matrix}60\\4\frac{1}{2}\end{matrix}\right\}:\left.\begin{matrix}x\\\frac{3}{4}\end{matrix}\right\}::1:1$ Ans. 360 yd.; **69.** $7\frac{1}{113}$ sec.; **70.** 174 da.; **71.** 338.375 bu.; **72.** $6\frac{1}{4}$%; **73.** $16\frac{2}{3}$%; **74.** The former, $66\frac{2}{3}$% better; **75.** 200 shares cost $2,400; **76.** $1,600; **77.** $21\frac{3}{7}$%; **78.** $17,762\frac{1}{2}$ bu.; **79.** $4,176.90; **80.** $1,560.50; **81.** \$$1,850.12\frac{1}{2}$; **82.** $15,000.00; **83.** $226.71; **84.** $125.84.